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Part I.: Exercise Manual.
Part II.: Examination Manual.

BOSTON, U.S.A.:
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PART FIRST.

ANSWERS TO EXERCISE MANUAL.

Ex. 2. Page 4.

1. $a^3 + 6a^2b + 6ab^2 + b^3$. 3. $2a^2b^2c$.

2. $-5x^4 - 4ax^3 + cx + 6d$. 4. $3a\sqrt{x} + 9y$.

5. $-31a^2 - 28\sqrt{x} + 16 - 15\sqrt{ax}$.

6. $15a^m + 33b^n - 29c^p + 37d^q$.

7. $3\frac{49}{120}a^2b + 8\frac{2}{15}a^3bc - 29\frac{7}{15}b^2c^4 + 11\frac{109}{630}$.

8. $9a - 4b - 17c - 12d + 12e$.

9. $6a^3 - 2a^2b - 3ab^2 - 22ac^2 - 7b^3 - c^3$.

10. $8a^3 + 2a^2b + 12ab^2 + b^3$.

11. $85x^2y^3z^4$.

12. $7a^m + 13a^{m-1}b + 20a^{m-2}b^2 + 13a^{m-3}b^3 - 8b^4$.

13. $(a - b - d + f + m - 3)x^3 + (7a + b - c + d - n + 5)x^2$
$- (c + 3d + g - p - 11)x$.

14. $(6m - p + 9)x^4y^3 - (2m - n + 1)x^3y^2 + (a + p - 1)x^2y$
$- (a - 10)x$.

15. $(3bc - 4ab + m + 1)x^p + (3ab + am + 11)y^q$
$- (a + 3b - 3m + mn - 1)z^k + (a + b + n + 7)t^m$.

16. $(a^2 + ab + 5)m^{p+2} + (3bc^2 - 3b - c)n^{q-1}$
$+ (3c^2d^2 - k)u^{r-3} + (a - 2fg - f)v^s$.

Ex. 3. Page 6.

1. $m^2 - 2n^2$.
2. $-n^2$.
3. $2n^2 - m^2$.
4. n^2.
5. $-2m^2 - n^2$.
6. $y^p - 2x^n$.
7. $-x^n$.
8. $2x^n - y^p$.
9. $x^n + 2y^p$.
10. $x^n + 2y^p$.
11. $-(m+n)$.
12. $m - n$.
13. $m - n$.
14. $2n - m$.
15. $5(m+n)$.
16. $a - 8b$.
17. $4(a-b)$.
18. $2a + b$.
19. $a - b$.
20. $3(a-b)$.
21. $-\frac{1}{2}$.
22. $\frac{8}{5}a - \frac{6}{7}b$.
23. $\frac{8}{9}y$.
24. $1\frac{2}{9}$.
25. $-\frac{1}{6}a + \frac{3}{40}b$.
26. $1\frac{1}{8}a - \frac{5}{6}b$.
27. $\frac{5}{6}x - 1\frac{13}{24}y$.
28. $\frac{4}{5}x^3 + \frac{x^2}{21}$.
29. $5\frac{1}{2}a^2 - 6\frac{1}{4}b^2$.
30. $a - 1\frac{5}{6}b$.
31. $n + 1$.
32. $n - 1$.
33. $-(x+1)$.
34. $a + x$.
35. $b - c$.
36. $a - b + 5$.
37. $a + b - 10$.
38. $5(a-b)$.
39. $-2y$.
40. $8x - 7y$.
41. $3x^2 + 3xy + y^2$.
42. $6a^2b + 2b^3$.
43. $-8(a^3b + ab^3)$.
44. $2x + 6a + 13b - 7c - 13$.
45. $-2x^4 + 2ax^3 + 2bx^2 + 6cx - 5d$.
46. $73x^4 - 114x^3y - 20x^2y^2 + 34xy^3 + 37y^4$.
47. $3x^2 + 15xy - y^2 - 17xyz - yz - 5z^2$.
48. $-3a^2x^2 + 9axy - 7aby^2$.
49. $19a^m - 17b^n - 2c^p + 10d^q$.
50. $\frac{1}{2}x^2 + 4ax - 1\frac{5}{6}a^2$.
51. $-2\frac{1}{4}x - 2\frac{1}{3}a + \frac{14}{15}b + \frac{1}{4}y$.
52. $\frac{7}{4}a - 19\frac{1}{5} + \frac{1}{4}c$.
53. $-2\frac{1}{2}y - 5\frac{1}{4}a - \frac{1}{12}x + \frac{1}{3}b$.
54. $2m^2$.
55. $(1-3a)x^4 + 2ax^3 + 2(2-b)x^2 - 3cx - 5d$.

56. $(a-e)x^3+(a+b)x^2-(b-c)x+d-4.$
57. $(a+b)x^3+(bc^2+gx^2)y-(c+f)xy^2-(d+e)y^3.$
58. $2a^2xy^3+(9c-4b)xy^2+(9a-5b^2)x^2y+5(a+3)y^4.$
59. $(m+1)x^5-(4mn-7p)x^4y+(3m-7ab)x^3y^2-7(m+1)x^2y^3.$
60. $b(a-c)x^5+c(a-b)x^4+3d(c+2e)x^3-2f(d+g)x^2.$
61. $3m(n-3)yz+7q(p+2)y^2z^2+4a(b+2)y^3z^3.$
62. 0.
63. $2(a^2+b^2+c^2).$
64. $2(a^2+c^2-b^2).$
65. $2(b^2+c^2-a^2).$
66. 0. 67. $30a^2-18ab+9b^2.$
68. $-2a^2+8ab-7b^2.$
69. $6a^2-4ab-7b^2.$
70. $-10a^2-2ab+11b^2.$
71. $16a^2-2ab-b^2.$
72. $-17a^2+6ab+6b^2.$
73. $a^3+13a^2b-9b^3.$
74. $-20a^3+14a^2b+8b^3.$

Ex. 4. Page 9.

1. $6x^3-12xy+3xz.$
2. $a^5b^2+2a^4b^3-a^3b^4.$
3. $-4a^3d-2a^3c^3d+3a^2cd^5+a^2d.$
4. $-a^2b^2c+\frac{8}{3}a^2c^4-\frac{4}{15}a^3cd^2+2a^2c.$
5. $-\frac{5}{2}b^4c^3+b^3c^6-2b^2c^4.$
6. $-3x^4y^2+9x^3y^3-9x^2y^4+6xy^2.$
7. $6a^2x^3-4a^2x^2y+8a^2xy^2.$
8. $8a^3b^3-20a^2b^4+28ab^5.$
9. $-a^3x+a^2x^2-ax^3.$
10. $x^4y-3x^3y^2+3x^2y^3-xy^4.$
11. $-3x^2y^{6-3n}+\frac{8}{7}x^{3-2m}y-x^3y^6+x^{3-2m}y^{8-3n}+3x^{3-2m}y^{6-3n}.$
12. $\frac{5}{6}ab^2-\frac{5}{2}a^5b^5+\frac{25}{6}a^4b^7-\frac{5}{8}a^{1-x}b^{2-2y}.$
13. $900c^3d^{3-2x}-504c^4d^{4-2x}-1440c^{4+p}d^{3-x}.$
14. $1350m^3y^p-1800m^{1+x}y^{1+p}+75m^{5-x}y^{p-4}.$

15. $-1700b^{3n-m+4}d^{3p}+1350b^{n+3}d^{3p+q+2}-25b^{n-2m+5}d^{2p+3q-1}$.

16. $1170d^{1+3y}-1620d^{3}+1800d^{2+5y}$.

17. $600x-1050x^{m-5n+4}+625x^{2m}$.

18. $42y^{p}-39y^{3-4m}+3y^{4p+1}$. 21. $4x^2-11xy-45y^2$.

19. $3a^2-7ab+4b^2$. 22. $6a^2-7ab+2b^2$.

20. $12b^2+bc-20c^2$. 23. $5a^4+7a^3b-8a^2b^2-4ab^3$.

24. $16x^4-32x^3y+16x^2y^2-8xy^3+3y^4$.

25. $3a^4-26a^3b+37a^2b^2-14ab^3$.

26. $10a^5-9a^4x+9a^3x^2-3a^2x^3+ax^4$.

27. $15x^4-23x^3+27x^2+9x-28$.

28. $x^4+4x^3-19x^2+64x-35$.

29. $6a^5b-7a^4b^2-11a^3b^3+9a^2b^4-5ab^5$.

30. $x^5-10ax^4+40a^2x^3-80a^3x^2+80a^4x-32a^5$.

31. $x^6-5x^4+3x^3+6x^2-7x+2$.

32. $x^7-9x^5y^2+7x^4y^3+13x^3y^4-19x^2y^5+8xy^6-y^7$.

33. $x^4-3ax^3-9a^2x^2-19a^3x-6a^4$.

34. $4x^8-16x^6-16x^5+12x^4+32x^3+24x^2+8x+1$.

35. $x^7-7x^6y+21x^5y^2-35x^4y^3+35x^3y^4-21x^2y^5+7xy^6-y^7$.

36. $a^3-6a^2b+9a^2c+12ab^2-36abc+27ac^2-8b^3$
$+36b^2c-54bc^2+27c^3$.

37. $6x^6+3x^5y-15x^4y^2-12x^3y^3+51x^2y^4-45xy^5+12y^6$.

38. $-6a^5x+22a^4x^2-34a^3x^3+54a^2x^4-68ax^5+32x^6$.

39. $2by^8-11b^2y^7+28b^3y^6-39b^4y^5+44b^5y^4-30b^6y^3$.

40. $6c^5-50c^4z+58c^3z^2-40c^2z^3+38cz^4+132z^5$.

41. $a^6-10a^5b+29a^4b^2-24a^3b^3-14a^2b^4+22ab^5-4b^6$.

42. $x^6-2x^5y-x^4y^2+4x^3y^3-x^2y^4-2xy^5+y^6$.

43. $6a^6b^3-27a^5b^4+44a^4b^5-30a^3b^6+3a^2b^7+2ab^8$.

44. $3x^7-11x^6+7x^5+11x^4-2x^3+x^2-28x+15$.

45. $35a^2+11a^3-15a^4+18a^5-68a^6+28a^7$.

46. $10p^5 + 16p^4q - 25p^3q^2 - 3p^2q^3 + 5pq^4 + q^5.$

47. $2a^7 - 5a^6b + 9a^5b^2 - 9a^4b^3 + 7a^3b^4 - 5a^2b^5 + 2ab^6 - b^7.$

48. $45c^{14} - 2c^{12} + c^{10} - 6c^8 - 131c^6 + 150c^4 - 52c^2.$

49. $a^6 - 15a^5b + 72a^4b^2 - 95a^3b^3 - 72a^2b^4 - 15ab^5 - b^6.$

50. $x^{12} - 12x^{11} + 46x^{10} - 36x^9 - 111x^8 + 72x^7 + 184x^6$
$+ 96x^5 + 16x^4.$

51. $14d^8 - d^7e - 7d^6e^2 - 485d^5e^3 + 50d^4e^4 - 436d^3e^5$
$+ 630d^2e^6 + 548de^7 - 16e^8.$

52. $3a^5 - 7a^4x + 8a^3x^2 - 8a^2x^3 + 3\frac{1}{3}ax^4 - x^5.$

53. $50c^7 - 82c^6 - 61\frac{1}{10}c^5 + 5990\frac{7}{10}c^4 + 1761\frac{1}{20}c^3 - 155\frac{3}{4}c^2$
$- 59\frac{1}{5}c + 4.$

54. $8d^8 - 33cd^7 - 32\frac{4}{9}c^2d^6 - 70\frac{1}{9}c^3d^5 - 56\frac{1}{27}c^4d^4 + 2\frac{5}{12}c^5d^3$
$+ 8\frac{35}{36}c^6d^2 - \frac{1}{4}c^7d.$

55. $-6p^8 + 201p^7q - 116p^6q^2 - 649p^5q^3 + 448p^4q^4$
$+ 187\frac{7}{8}p^3q^5 - 328p^2q^6 + 179\frac{1}{2}pq^7 - 5\frac{1}{2}q^8.$

56. $168c^9 - 211\frac{1}{3}c^8d + 34c^7d^2 + 82c^6d^3 - 139\frac{2}{3}c^5d^4$
$+ 72\frac{11}{18}c^4d^5 + \frac{2}{3}c^3d^6 - \frac{1}{6}c^2d^7.$

57. $\frac{c^6}{27} - \frac{c^5}{12} - \frac{5c^4}{48} + \frac{15c^3}{64} + \frac{5c^2}{32} - \frac{3c}{16} - \frac{1}{8}.$

58. $\frac{x^2}{4} - \frac{x^3}{3} + \frac{13x^4}{36} - \frac{x^5}{6} + \frac{x^6}{16}.$

59. $0.2m^6 - 0.46m^5n + 0.32m^4n^2 - 0.08m^3n^3 - 2m^2n^4$
$+ 0.2mn^5 + 1.12n^6.$

60. $14x^5 - 6x^4y - 7x^3 + 2x^3y^2 + 10x^2y - 4xy^2 + y^3.$

61. $14x^9 + 21x^8 - 6x^7 + 38x^6 + 7x^5 + 4x^4 - 30x^3 + 26x^2$
$- 36x - 20.$

62. $x^7 - 2\frac{1}{6}x^6y + 7x^5y^2 - 10\frac{1}{3}x^4y^3 + 12\frac{2}{3}x^3y^4 - 8\frac{2}{3}x^2y^5$
$+ 6xy^6 - 2y^7.$

63. $9b^6 - 30b^5d + 40b^4d^2 - 46b^3d^3 + 44b^2d^4 - 26bd^5 + 3d^6.$

64. $8a^9 - 36a^8b - 30a^7b^2 + 225a^6b^3 + 105a^5b^4 - 441a^4b^5$
$- 343a^3b^6$.

65. $27p^7 - 21p^6x + 18p^5x^2 + 74p^4x^3 - 29p^3x^4 + 23p^2x^5$
$+ 72px^6$.

66. $8b^6 + 10b^5 + 4b^4 - 21b^3 - 6b^2 + 5b$.

67. $3 + 7a - 8a^2 + 19a^3 - 10a^4 + 4a^5$.

68. $a^2b + a^2c + abc - ab^2 + ac^2 - b^2c + bc^2$.

69. $1 - 2x + 4xy - 2xy^2 + x^2 - 2x^2y + x^2y^2 - 2y + y^2$.

70. $b^2cx^2z - bc^2x^2y + 3abcxyz - ac^2xy^2 - ab^2xz^2 + a^2cy^2z$
$- a^2byz^2$.

71. $x^4 - (a - b)x^3 - (ab - b + c)x^2 + (ac + b^2)x - bc$.

72. $x^4 - (a - p)x^3 - (b^2 + ap + q^2)x^2 + (aq^2 - b^2p)x + b^2q^2$.

73. $x^4 - (3a - 5b)x^3 - (6a^2 + 15ab - 8b^2)x^2$
$- 6ab(5a + 4b)x - 48a^2b^2$.

74. $x^5 - (a + m)x^4 + (am + b + n)x^3 - (an + bm + c)x^2$
$+ (bn + cm)x - cn$.

75. $x^5 - (a - m)x^4 + (b + n - am)x^3 - (an - bm + c)x^2$
$+ (bn - cm)x - cn$.

76. $cx^5 - (b + c)x^4 + (a + b - c)x^3 - (a - b + 1)x^2$
$+ (1 - a)x + 1$.

77. $adz^4 + (ae + bd)z^3 + (af + be + cd)z^2 + (bf + ce)z + cf$.

78. $mrz^6 - (nr - ms)z^5 + (mp - ns - pr)z^4$
$- (mu + np + ps - qr)z^3 + (nu - p^2 + qs)z^2$
$+ p(q + u)z - qu$.

79. $3ma^5 + (9m - 4n)a^4 - (12m + 12n - p)a^3 + (16n + 3p)a^2$
$- 4pa$.

80. $8amx^5 + 2(5an - 6bm)x^4 - (4ap + 15bn - 16dm)x^3$
$- 2(av - 3bp - 10dn)x^2 + (3bv - 8dp)x - 4dv$.

81. $a^m + 3a^{m+1} - a^{m+2} - 3a^{m+3} - a - 1$.

82. $a^{n+1} - 5a^n + 9a^{n-1} - 4a^{n-2} - a^{n-3}$.

83. $a^{n+1} - 2a^n b + a^{n-1}b^n + a^{n-1}b^2 - a^{n-2}b^{n+1}$.

84. $x^n + x^{n-5}y^5$.

85. $2a^{p+3} - 7a^{p+2} + 12a^{p+1} - 19a^p + 20a^{p-1} - 11a^{p-2} + 5a^{p-3}$.

86. $a^{3n+3} - 2a^{2n+3} - 2a^{2n+2} + 2a^{2n+1} + a^{n+3} + 2a^{n+2} - a^{n+1}$
$- 2a^n + a^{n-1}$.

87. $6x^{8n+2} - 23x^{7n+1} + 18x^{6n} - x^{5n-1} - 3x^{4n-2} + 4x^{3n-3} - x^{2n-4}$.

88. $m^{p+r} - 11m^{p+r-2}n^2 - 13m^{p+r-3}n^3 + 3m^{p+r-5}n^5$.

89. $e^{2m} - 2e^{2m-1}d + 3e^{2m-2}d^2 - 4e^{2m-3}d^3 + 3e^{2m-4}d^4 - 2e^{2m-5}d^5$
$+ e^{2m-6}d^6$.

90. $4x^{2r+7}y^{1-3q} - 9x^{2r+5}y^{1-q} + 22x^{2r+4}y - x^{2r+3}y^{1+q} + 16x^{2r+1}y^{1+3q}$.

91. $2a^9 b - 18a^4 b^6 + 6a^3 b^7 - 18a^2 b^8 + 4ab^9$.

92. $2u^{q+1} - 6u^{q+2} + 2u^{q+3} - 4u^{q+4} + 3u^{2q+1} - 9u^{2q+2}$
$+ 3u^{2q+3} - 6u^{2q+4} - 4u^{3q+1} + 12u^{3q+2} - 4u^{3q+3} + 8u^{3q+4}$.

93. $-8x^{2p+4}y^{2r-1} - 10x^{2p+3}y^{2r} + 19x^{2p+2}y^{2r+1} + 16x^{2p+1}y^{2r+2}$
$- 5x^{2p}y^{2r+3}$.

94. $a^{2p+2} - 4a^{2p} + 12a^{2p-1} - 9a^{2p-2}$.

95. $9y^{m+p-3} - 34y^{m+p-1} + 29y^{m+p+1} - y^{m+p+3}$.

Ex. 5. Page 13.

1. $5a^5 + 5$.
2. $7x^2 + 8x + 1$.
3. $z^3 + bz^2 + b^2 z + b^3$.
4. $2a^2 + 1$.
5. $6a^2 - a - 1$.
6. $x^3 + 3x^2 - x - 3$.
7. $b^3 + b^2 - 5b + 3$.
8. $12m^3 + 20m^2 n + 8mn^2$.
9. $d^4 - 2d^3 + 3d^2 - 2d + 1$.
10. $2m^2 + 1$.
11. $p^2 + p + 1$.
12. $3a^2 x - 4ax^2 + x^3$.
13. $y^2 - 5y + 6$.
14. $2y^3 - 4y^2 + y - 1$.
15. $2x - 7y$.
16. $2a^2 + 5a - 7$.
17. $3a^2 - 2ab - b^2$.
18. $3x^2 - 2ax + 5a^2$.
19. $a^4 + a^2 x^2 + x^4$.
20. $5x + 2$.
21. $3x^2 + 4xy + y^2$.
22. $x^2 - a^2$.

23. $x^2 + 5xy + 3y^2$.

24. $2x^2 + 5xy - y^2$.

25. $x^3 - 7x + 5$.

26. $x^2 - 3x + 1$.

27. $3x^2 + 2xy - y^2$.

28. $x^2 + 5x + 6$.

29. $5x^4 - 2x^3y + 4x^2y^2$.

30. $7a^2 - 3ab + 2b^2$.

31. $\frac{4}{9}b^2 - \frac{3}{5}bd + \frac{1}{4}d^2$.

32. $\frac{1}{6}d^3 - \frac{2}{3}d^2 + \frac{1}{2}d$.

33. $\frac{3}{8}a^3 - \frac{2}{3}a^2b + \frac{1}{2}ab^2$.

34. $\frac{1}{7}m^2 + \frac{3}{4}mn - \frac{2}{3}n^2$.

35. $\frac{2}{5}x^2 + \frac{3}{4}xy - \frac{1}{2}y^2$.

36. $\frac{2}{3}a^3 - \frac{1}{2}a^2b + \frac{5}{6}ab^2 - \frac{1}{3}b^3$.

37. $x^3 - \frac{3}{5}x^2 - 2x + \frac{1}{8}$.

38. $\frac{1}{2}a^3 + \frac{2}{3}a^2b - \frac{1}{4}ab^2 + \frac{3}{4}b^3$.

39. $1\frac{1}{5}y^4 - \frac{3}{4}y^3z - \frac{3}{5}y^2z^2 + \frac{1}{2}yz^3 + \frac{3}{8}z^4$.

40. $2\frac{1}{2}z^5 + 3\frac{1}{4}z^4 - 6\frac{1}{2}z^3 + \frac{3}{5}z^2 - \frac{1}{3}z + \frac{2}{7}$.

41. $-\frac{5}{2}a^3 - \frac{3}{4}a^2b + \frac{1}{2}ab^2 + 6b^3$.

42. $-(4a + x)$. 43. $8x - 5$.

44. $x^4 - 3x^3y - 5x^2y^2 + 7xy^3 - 2y^4$.

45. $4x^3 - 5x^2y - 8xy^2 + 2y^3$.

46. $a^4 - 3a^3b + 4a^2b^2 - 2ab^3 + 4b^4$.

47. $3x - 8x^2y + 7x^3y^2 - 5x^4y^3 + 7x^5y^4$.

48. $5y + 7y^2z - 9y^3z^2 + y^4z^3$.

49. $\frac{5}{6}a^3 - \frac{2}{3}a^2x + \frac{1}{4}ax^2 - \frac{4}{5}x^3$.

50. $a^4 - 7a^2b^2 + 6ab^3 - b^4$.

51. $2a^2 - 6a + 2$.

52. $7x^2 + 3xy - 9y^2$.

53. $2x^3y + 2x^2y^2 - xy^3$.

54. $2x^2 + xy - 3y^2$.

55. $x^3 + 3x^2 + x - 2$.

56. $x^5y^5 + 2x^4y^4 + 4x^3y^3 + 8x^2y^2 + 16xy + 32$.

57. $y^9 + y^7 + y^6 + y^5 + y^4 + y^3 + y^2 + 1$.

58. $a - b$.

59. $3x^2 - 3xy + 4y^2$.

60. $2a^2 + 3ab - b^2$.

61. $(a + b)^3$.

62. $a^2 + 2ab + b^2 + c$.

63. $2x^2 + 3xy + y^2 - 2$.

64. $a - 2a^2 + 3a^3$.

65. $a^m + 2a^{m+1} - 4a^{m+2} + 5a^{m+3}$.

66. $3d^x - 4d^{x-1} + 5d^{x-2} - d^{x-3}$.

67. $4h^4 - 2h^3 - 3h^2$.

68. $2x^{m-1}+3x^{m-2}-4x^{m-3}$. 72. $x^{(m-1)n}-y^{(n-1)m}$.

69. $3b^{x}-4b^{x-1}+3b^{x-2}-b^{x-3}$. 73. $x^{2n}-2x^{n}y^{n}+y^{2n}$.

70. $3x^{4n+1}-4x^{3n}+2x^{2n-1}-x^{n-2}$. 74. $a^{n}b^{n}-7a^{n-1}b^{2n}$.

71. $a^{m}+3a^{m-1}b^{n}-6a^{m-2}b^{2n}$. 75. $1-x+x^2-x^3+x^4-\cdots\cdots$

76. $1+x+x^2+x^3+x^4+\cdots\cdots$

77. $1-\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x^3}+\frac{1}{x^4}-\cdots\cdots$

78. $\frac{1}{x}+\frac{1}{x^3}+\frac{1}{x^5}+\frac{1}{x^7}+\frac{1}{x^9}+\cdots\cdots$

79. $a-ax+ax^2-ax^3+ax^4-\cdots\cdots$

80. $m+my+my^2+my^3+my^4+\cdots\cdots$

81. $1+5x+15x^2+45x^3+135x^4+\cdots\cdots$

82. $1-4a+8a^2-16a^3+32a^4-\cdots\cdots$

83. $1+\frac{2y}{x}+\frac{2y^2}{x^2}+\frac{2y^3}{x^3}+\frac{2y^4}{x^4}+\cdots\cdots$

84. $1-(a+b)x+(a+b)bx^2-(a+b)b^2x^3+(a+b)b^3x^4-\cdots\cdots$

85. $\frac{a}{b}+\frac{a-b}{b^2y}+\frac{a-b}{b^3y^2}+\frac{a-b}{b^4y^3}+\frac{a-b}{b^5y^4}+\cdots\cdots$

or $\frac{a}{b}+\frac{a}{b^2y}+\frac{a}{b^3y^2}+\frac{a}{b^4y^3}+\cdots\cdots$

86. $1+2x+3x^2+4x^3+5x^4+\cdots\cdots$

87. $\frac{1}{x}+\frac{2}{x^2}+\frac{4}{x^3}+\frac{8}{x^4}+\frac{16}{x^5}+\cdots\cdots$

88. $\frac{1}{3}-\frac{x}{9}+\frac{x^2}{27}-\frac{x^3}{81}+\frac{x^4}{243}-\cdots\cdots$

89. $-\frac{1}{2}-\frac{x}{4}-\frac{x^2}{8}-\frac{x^3}{16}-\frac{x^4}{32}-\cdots\cdots$

90. $1+3x+5x^2+7x^3+9x^4+\cdots\cdots$

91. $1+\frac{x}{1} = 2$

$\frac{x^2}{1\times 2} = 0.5$

$\frac{x^3}{1\times 2\times 3} = 0.1666666$

$\frac{x^4}{1\times 2\times 3\times 4} = 0.0416666$

$\frac{x^5}{1\times 2 \cdots\cdots 5} = 0.0083333$

$\frac{x^6}{1\times 2 \cdots\cdots 6} = \underline{0.0013888}$

$\frac{x^7}{1\times 2 \cdots\cdots 7} = 0.0001984$

$\frac{x^8}{1\times 2 \cdots\cdots 8} = 0.0000248$

$\frac{x^9}{1\times 2 \cdots\cdots 9} = 0.0000027$

$\frac{x^{10}}{1\times 2 \cdots\cdots 10} = 0.0000003$

$\frac{x^{11}}{1\times 2 \cdots\cdots 11} = \underline{0.0000000}$

0.0002262

2.7180553

$\underline{0.0002262}$

$= 2.7182815 = 2.71828$ *Ans.*

92. $1+\frac{1}{5} = 1.2$

$\frac{1}{9} = 0.1111111$

$\frac{1}{13} = 0.0769230$

$\frac{1}{17} = 0.0588235$

$\frac{1}{21} = 0.0476190$

$\frac{1}{25} = 0.04$

$\frac{1}{29} = \underline{0.0344827}$

1.5689593

$-\underline{0.766911}$

$0.802048 \times 4 = 3.208192$ *Ans.*

$\frac{1}{3} = 0.333333$

$\frac{1}{7} = 0.142857$

$\frac{1}{11} = 0.090909$

$\frac{1}{15} = 0.066666$

$\frac{1}{19} = 0.052631$

$\frac{1}{23} = 0.043478$

$\frac{1}{27} = \underline{0.037037}$

0.766911

93. $\frac{a}{1} = 0.2$

$\frac{a^5}{5} = 0.000064$

$\frac{a^9}{9} = \underline{0.000000056}$

0.200064056

$-\underline{0.002668496}$

$= 0.19739556 = 0.19739$ *Ans.*

$\frac{a^3}{3} = 0.002666666$

$\frac{a^7}{7} = 0.000001828$

$\frac{a^{11}}{11} = \underline{0.000000002}$

0.002668496

94. $x = 1.5708$

$\frac{x^5}{1 \times 2 \cdots\cdots 5} = 0.079693558$

$\frac{x^9}{1 \times 2 \cdots\cdots 9} = 0.000160444$

$\frac{x^{13}}{1 \times 2 \cdots\cdots 13} = \underline{0.000000056}$

1.650654058

$-\underline{0.650654058}$

$= 1.0$ *Ans.*

$\frac{x^3}{1 \times 2 \times 3} = 0.6459686291$

$\frac{x^7}{1 \times 2 \cdots\cdots 7} = 0.0046818307$

$\frac{x^{11}}{1 \times 2 \cdots\cdots 11} = \underline{0.0000035989}$

0.6506540587

95. $\frac{1}{2n+1} = 0.3333333333$

$\frac{1}{3(2n+1)^3} = 0.0123456790$

$\frac{1}{5(2n+1)^5} = 0.0008230453$

$\frac{1}{7(2n+1)^7} = 0.0000653210$

$\frac{1}{9(2n+1)^9} = 0.0000056450$

$\frac{1}{11(2n+1)^{11}} = 0.0000005131$

$\frac{1}{13(2n+1)^{13}} = 0.0000000482$

$\frac{1}{15(2n+1)^{15}} = \underline{0.0000000046}$

0.3465735895×2

$= 0.69314$ *Ans.*

Ex. 6. Page 17.

1. $4a-2c$.
2. $4a$.
3. $8a-9b-8c$.
4. $3a-3x+c$.
5. $21-11x$.
6. $4c$.
7. $-8x^3-8x$.
8. $2bce+2bef$.
9. $8+3x$.
10. $6x-5y$.
11. $12a-3x$.
12. $(2a^2-a-3)b^3+(7a^2-9a-1)b^2+(4a^2-8a+9)b$
$-(4a^2-12a+5)$.
13. $(a^3-b^3)m^5+(a^2+2ab+3b^2)m^4$
$-(a^2b-ab^2+2a+2b+a^2+ab+b^2)m^3$
$+(a+b-2ab)m^2+abm$.
14. $2(a-b)x^5-(a^2-2ab-b^2)x^4+2(a+b-ab^2)x^3$
$-(2a^2+2b^2-a^2b^2)x^2+2ab^2x-(a^2-b^2)$.
15. $x^6+2x^5y-(a^2-2a+3)x^4y^2+(a-6)x^3y^3$
$-(7a-2)x^2y^4-(a^2-4)xy^5$.
16. $(b^2-a^2)m^5-(a^2+3b^2-4ab)m^4+2(a^2-3ab+2b^2)m^3$
$-2(3a^2-5ab+2b^2)m^2+4a(a-b)m$.
17. $(a^2-4b^2+2a+1)x^4+4b(4a+1)x^3$
$-(7a^2+6ab-a+1)x^2+(6a^2+10ab-3a-2b-4b^2)x$
$-2b(2a-1)$.
18. $(b^4-2b^2+1)x^4-2(b^3-2b^2-b+2)x^3$
$-(2b^3-3b^2+2b-2)x^2+2(b^2-3b+2)x+(1-b)^2$.
19. $729a^6-243(1-m)a^5-54(4-5m^2+m)a^4$
$+(53-51m-57m^2+55m^3)a^3$
$+6(4+m-9m^2-m^3+5m^4)a^2$
$-3(1-m-2m^2+2m^3+m^4-m^5)a-(1-m^2)^3$.

NOTE. In this example put x for $1-m$ and y for $1-m^2$. The multiplication then gives $729a^6-243xa^5-(243y-27x^2)a^4+(54xy-x^3)a^3+(27y^2-3x^2y)a^2-3xy^2a-y^3$. Then, x and y are replaced by their values.

20. $(4a^2-9c^2)y^4-2acy^3-(a^2-16ac-c^2)y^2$
$+2(2a^2+c^2)y-(4a^2-c^2)$.

21. $x^5+2ax^4+(a^2-a+3)x^3+(a^3-4a^2+7a-9)x^2$
$-(a^3-9a^2+22a-10)x$
$+(a^5-8a^4+24a^3-36a^2+29a-7)$.

22. $y^5+2by^4+(a^2-2ab+b^2)y^3+(a^3-3a^2b+3ab^2-b^3)y^2$
$+2(a^3b-3a^2b^2+3ab^3-b^4)y$
$+(a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5)$.

23. $2(x^2+y^2-z^2)$.

24. $-2(10a^4-15a^3+7a^2b+12ab-12b^2)$.

25. $3a^2b-2ab^2$. **26.** 0. **29.** 0.

27. $-10(a^3-a^2b-ab^2+b^3)$. **30.** $37b^2-ab-2b+2a-9a^2$.

28. $12y^2-28xy$. **31.** $-27x^2-14xy+55y^2$.

32. $-5a^3+9a^2c+3a^2b-9abc+2ab^2$.

33. $10a^4b-19a^3b^2+8a^2b^3+ab^4-a^2b+2ab^2-b^3$.

34. $(x+y)^2(2a^2x+x+y)$
$=(2a^2+1)x^3+(4a^2y+3y)x^2+(2a^2y^2+3y^2)x+y^3$.

35. a^2+ax-3. **38.** $a+2b-c+3d$.

36. a^2-pa+q. **39.** $x^2-(y-1)x+(y^2+y+1)$.

37. $z^2+(a-b)z-a^2$. **40.** $(a-1)x+a^2$.

41. $(x^2+x+1)y-(x+1)$.

42. $x^3+x^2(a-2y)+x(y-2a)y+ay^2$.

43. $z^2+(m+n)z-mn$. **45.** $4(a+b)^2-6(a+b)x+8x^2$.

44. my^3+ny^2+oy+p. **46.** $4x^3-5x^2y-3xz^2+6z^3$.

Ex. 7. Page 20.

1. $x^2+2xy+y^2$. **6.** $4x^2-4ax+a^2$.

2. $b^2-2bc+c^2$. **7.** $27a^3-27a^2b+9ab^2-b^3$.

3. $m^3+3m^2n+3mn^2+n^3$. **8.** $16a^2+25b^2-40ab$.

4. $a^3-3a^2b+3ab^2-b^3$. **9.** $9x^2+30xy+25y^2$.

5. $m^2+4mn+4n^2$. **10.** $64z^2-80z+25$.

11. $9 + 12d + 4d^2$.

12. $16m^2 - 40mn + 25n^2$.

13. $1 + 4x^2 + 4x^4$.

14. $9 + 30m^5 + 25m^{10}$.

15. $16a^2 - 24a^3 + 9a^4$.

16. $4x^4 + 20x^2 + 25$.

17. $49c^2 - 56cd + 16d^2$.

18. $64m^2 + 64mn + 16n^2$.

19. $81p^2 - 54pq + 9q^2$.

20. $25a^6 - 20a^3b^4 + 4b^8$.

21. $a^6 - 6a^4b + 12a^2b^2 - 8b^3$.

22. $y^3 + 6y^2z + 12yz^2 + 8z^3$.

23. $8u^3 + 12u^2v + 6uv^2 + v^3$.

24. $125a^3 - 75a^2b + 15ab^2 - b^3$.

25. $x^6 + 3x^4y^3 + 3x^2y^6 + y^9$.

26. $729m^9 - 1215m^6n + 675m^3n^2 - 125n^3$.

27. $b^3c^6 - 3ab^4c^4 + 3a^2b^5c^2 - a^3b^6$.

28. $m^6n^3 + 3m^4n^2p^2q + 3m^2np^4q^2 + p^6q^3$.

29. $1000 - 300x^2 + 30x^4 - x^6$.

30. $216 + 540a^3 + 450a^6 + 125a^9$.

31. $343d^6 - 294d^4 + 84d^2 - 8$.

32. $512z^{12} - 1728z^8 + 1944z^4 - 729$.

33. $m^2 - n^2$.

34. $1 - x^2$.

35. $9 - y^2$.

36. $9x^2 - 4y^2$.

37. $9d^2y^2 - 1$.

38. $25x^4 - 4y^6$.

39. $9a^2b^4 - 25a^4b^2$.

40. $16x^4y^2 - 36x^2y^4$.

41. $a^4 - 9x^2$.

42. $25 - b^2x^6$.

43. $49n^8 - 36m^2$.

44. $144 - a^2b^2$.

45. $a^2x^8 - a^8x^2$.

46. $a^2 + 2ab + b^2 - c^2$.

47. $a^2 - 2ab + b^2 - c^2$.

48. $x^4 + x^2y^2 + y^4$.

49. $4x^2 - 4xy + y^2 - 9z^2$.

50. $x^2 + 6xz + 9z^2 - 4y^2$.

51. $a^2 + 6ac + 9c^2 - 4b^2 - 4bd - d^2$.

52. $1 + 2x + x^2 - 4x^4 - 12x^5 - 9x^6$.

53. $4 + 12a^3 + 9a^6 - a^4 - 2a^2d^2 - d^4$.

54. $a^4 - x^4$.

55. $m^2 + 3m + 2$.

56. $b^4 + 6b^2 + 8$.

57. $x^2 + 5x + 6$.

58. $9y^4 + 60y^2 + 96$.

59. $x^2 + 3x - 10$.

60. $9a^2 - 9a - 28$.

61. $4a^2 - 12a - 55$.

62. $16x^2 - 48x + 27$.

63. $25d^2 - 85d + 66$.

64. $m^2 + (n + 2)m + 2n$.

65. $1 + 3a + b + 3ab$.

66. $16a^2 + 4a(b + 1) + b$.

67. $9a^2 + 3a(4 - n) - 4n$.

68. $25x^2 - 5x(a - 1) - a$.

69. $9x^4 + 3x^2(p - m) - mp$.

70. $25y^8 + 5y^4(3x + 2a) + 6ax$.

71. $m^6 + 2m^3(1 - x) - 4x$.

72. $x^3 + y^9$.

73. $y^3 + 27z^3$.

74. $m^3 - n^3$.

75. $8 - a^3$.

76. $d^3 + 125$.

77. $a^9 - 27$.

78. $64m^6 - y^{12}$.

79. $343x^9 - 512y^3$.

80. $27m^3 - a^3$.

81. $4a^2 + 16b^2 + 16ab + 25c^2 + 4d^2 - 20cd - 20ac - 40bc + 8ad + 16bd$.

82. $9x^2 + 4y^2 - 12xy + 16z^2 + 25 - 40z + 24xz - 16yz - 30x + 20y$.

83. $4p^4 + 9x^8 + 12p^2x^4 + 4x^2y^2 + y^4 + 4xy^3 - 8p^2xy - 12x^5y - 4p^2y^2 - 6x^4y^2$.

84. $a^2 + 4b^4 + 4ab^2 + 9c^6 + 16d^8 + 24c^3d^4 + 6ac^3 + 12b^2c^3 + 8ad^4 + 16b^2d^4$.

85. $a^4 + 25b^6 - 10a^2b^3 + 4a^2 + 9b^4 - 12ab^2 + 4a^3 - 20ab^3 - 6a^2b^2 + 30b^5$.

86. $\frac{1}{4}x^4 + 16y^2 - 4x^2y + \frac{4}{9}y^4 + 36z^6 + 8y^2z^3 + \frac{2}{3}x^2y^2 - \frac{16}{3}y^3 + 6x^2z^3 - 48yz^3$.

87. $\frac{4}{25}a^2 + \frac{9}{4}a^4 - \frac{6}{5}a^3 + 64b^4 + \frac{1}{4}b^2c^2 - 8b^3c - \frac{32}{5}ab^2 + 24a^2b^2 + \frac{2}{5}abc - \frac{3}{2}a^2bc$.

88. $0.36m^4 + 0.25n^2 - 0.6m^2n + 0.64p^6 + 9x^{10} - 4.8p^3x^5 + 0.96m^2p^3 - 0.8np^3 - 3.6m^2x^5 + 3nx^5$.

89. $-3.91a^2x^2+x^4+0.6ax^3+4a^4+16x^2-16a^2x-1.2a^3x$
$+2.4ax^2+8x^3$.

90. $x^6-2ax^5+5a^2x^4-6a^3x^3+6a^4x^2-4a^5x+a^6$.

91. $m^6+6dm^5+15d^2m^4+20d^3m^3+15d^4m^2+6d^5m+d^6$.

92. $y^8-4xy^7+10x^2y^6-20x^3y^5+25x^4y^4-24x^5y^3+16x^6y^2$.

93. x^2-xy+y^2.

94. x^2+xy+y^2.

95. $a-y$.

96. $h+l$.

97. $1-x+x^2$.

98. y^2+y+1.

99. $u-v$.

100. $x+y$.

101. $a^3+a^2m+am^2+m^3$.

102. $a^3b^3+a^2b^2xy+abx^2y^2+x^3y^3$.

103. $h^5+h^4m+h^3m^2+h^2m^3+hm^4+m^5$.

104. $x^6+x^5y+x^4y^2+x^3y^3+x^2y^4+xy^5+y^6$.

105. $m^7-m^6n+m^5n^2-m^4n^3+m^3n^4-m^2n^5+mn^6-n^7$.

106. $a^8-a^7b+a^6b^2-a^5b^3+a^4b^4-a^3b^5+a^2b^6-ab^7+b^8$.

107. $2h+m$.

108. $1-x+x^2-x^3+x^4-x^5+x^6$.

109. x^2+x+1.

110. $b^4x^4-b^3cx^3y+b^2c^2x^2y^2-bc^3xy^3+c^4y^4$.

111. $8-4x+2x^2-x^3$.

112. $27+9y+3y^2+y^3$.

113. $16a^4-8a^3b+4a^2b^2-2ab^3+b^4$.

114. $9m^2+3mn+n^2$.

115. $4x-y$.

116. $125u^3+25u^2v+5uv^2+v^3$.

117. $a^5b^5+a^4b^4x+a^3b^3x^2+a^2b^2x^3+abx^4+x^5$.

118. $d^3-d^2xy+dx^2y^2-x^3y^3$.

119. $1-ay+a^2y^2-a^3y^3+a^4y^4$.

120. $a+b+c$.

121. $x+a-b$.

122. $a+b-c+d.$

123. $(x+y)^4-(x+y)^3z+(x+y)^2z^2-(x+y)z^3+z^4.$

124. $a^6-a^5(y+z)+a^4(y+z)^2-a^3(y+z)^3+a^2(y+z)^4$
$-a(y+z)^5+(y+z)^6.$

125. $(m+n)^2+(m+n)p+p^2.$

126. $(a-b)^3-(a-b)^2x+(a-b)x^2-x^3.$

127. $d^5+d^4(e-h)+d^3(e-h)^2+d^2(e-h)^3+d(e-h)^4$
$+(e-h)^5.$

128. $f^3-f^2(x-y)+f(x-y)^2-(x-y)^3.$

129. $m^3n^3+m^2n^2(x+y)+mn(x+y)^2+(x+y)^3.$

130. $\frac{1}{2}a-\frac{1}{3}d.$

131. $\frac{1}{4}x^2-\frac{1}{2}xy+y^2.$

132. $\frac{1}{81}a^4+\frac{1}{27}a^3m+\frac{1}{9}a^2m^2+\frac{1}{3}am^3+m^4.$

133. $\frac{1}{25}d^2-\frac{1}{20}dh+\frac{1}{16}h^2.$

134. $1+\frac{2}{3}x+\frac{4}{9}x^2.$

135. $x^4-x^2y+y^2.$

136. $a^8+a^6m^3+a^4m^6+a^2m^9+m^{12}.$

137. $y^9-y^6z+y^3z^2-z^3.$

138. $m^6+m^4n^3+m^2n^6+n^9.$

Ex. 8. Page 23.

1. $a(a+2b).$
2. $(3x-1)(x+1).$
3. $(6x+7a)(6x-7a).$
4. $(x^2+8)(x^2-8).$
5. $\left(\frac{x}{a}-\frac{a}{x}\right)\left(\frac{x}{a}-\frac{a}{x}\right).$
6. $(x-3)(x+2).$
7. $x(x+2)(x^2+x+1)(x^2+3x+3).$
8. $(x+1)(x^4-x^3+x^2-x+1).$
9. $(x+1)(x+7).$
10. $(2x+3)(2x+13).$

11. $(a+b+c-d)(a+b-c+d)(a-b+c+d)(c+d-a+b)$.

12. $(a-d+b-c)(a-d-b+c)$.

13. $(x+a)(x+b)(x+c)$.

14. $(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$.

15. $(x+y)[(a^2+ab+b^2)x+(a^2-ab+b^2)y]$.

16. $(a+2)(a-2)(a-2b+1)$.

17. $[(a+b)^2+4a^2]^2-[2a(a+b)]^2$.

18. $(x-7)(x+4)$.

19. $(1-10x)(1+3x)$.

20. $(x-2)(x+2)(x^2+2x+4)(x^2-2x+4)$.

21. $3a^2(a+2b)$.

22. $(x-2)(x+3)$.

23. $(3x+8)(5x-7)$.

24. $(9a^2b^2+16c^2)(3ab+4c)(3ab-4c)$.

25. $(x+2y)(x-2y)$.

26. $(x^2+\frac{10}{13}x+1)(x^2-\frac{10}{13}x+1)$.

27. $(2x-3)(3x-2)$.

28. $(2x-3y)(2x+3y)(4x^2-6xy+9y^2)(4x^2+6xy+9y^2)$.

29. $(x+1)(x+22)$.

30. $(3x+5y)(3x-5y)$.

31. $(x-1)(x-5)$.

32. $(x^2+\frac{1}{2})(x^2+\frac{1}{2})$.

33. $(x+12)(x+12)$.

34. $(x-2)(x^2+2x+4)$.

35. $7x^4(x^2-3)$.

36. $x(x-12)$.

37. $(x^2+2)(x^2-2)$.

38. $(x-7)(x-9)$.

39. $(a+b)(c-d)$.

40. $(1-8x)(1+12x)$.

41. $(2x+y-z)(2x-y+z)$.

42. $(x+3)(x^2-3x+9)$.

43. $(ab-1)(a^2b^2+ab+1)$.

44. $7xy(x+y)(x^2+xy+y^2)^2$.

45. $-4x^6(x-16)$.

46. $(x^n+ay^n)(x^n+by^n)$.

47. $(x+ab)(x-ab)(x+2a^2)(x-2a^2)$.

48. $(x+3y)(x^2+3xy+9y^2)$.

49. $(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$.

50. $(x-bc-ca)(x+ca+ab)$.

51. $(x+y+z)(x^2+y^2+z^2-yz-zx-xy)$.

52. $(x-y)[(a^2+ab+b^2)x+(a^2-ab+b^2)y]$.

53. $(x^2+yz)(x^4-4x^2yz+7y^2z^2)$.

54. $(a^2-b^2-c^2)(a^2-b^2+c^2)$.

55. $(x^2-ab)(x-a)(x-b)$.

56. $(a+b+c+d)(a+b+c+d)$.

57. $(x^2y^3+8z)(x^2y^3+12z)$.

58. $(ax-by)(ac-bd)$.

59. $(a+b-c)(a+b-c)$.

60. $(x-a)(x+2a+b)$.

61. $(6x^2+5)(6x^2+11)$.

62. $3(y+z)(z+x)(x+y)$.

63. Write the expression in the form:

$$x^4-xy^3-xy^3+y^4=x(x^3-y^3)-y^3(x-y)$$
$$=(x-y)(x^3+x^2y+xy^2-y^3).$$

64. $\left(x-\frac{1}{x}+\frac{1}{3}\right)\left(x-\frac{1}{x}-\frac{1}{3}\right)$.

65. $(x^2+1)(ax+1)$.

66. $(x^3+4y^2)(x^{12}-4x^9y^2+16x^6y^4-64x^3y^6+256y^8)$.

67. $3ab(a+b)$.

68. $(x^2+3x-1)(x^2-3x-1)$.

69. $5x(x^2-2x-1)$.

70. $\left(x^m+\frac{1}{x^m}\right)\left(x^m+\frac{1}{x^m}\right)$.

71. $\left(\frac{x}{a}+\frac{a}{x}\right)\left(\frac{x}{a}+\frac{2a}{x}\right)$.

72. $(x^2+y^2+xy)(x^2+y^2-xy)$

73. $(x-4y)(x-5y)$.

74. $-(x-3y)(x-3y)$.

75. $(a^4+9)(a^2+3)(a^2-3)$.

76. $a(a-4b)$.

77. $(x-1)(x^2+x+1)$
(x^6+2x^3+3).

78. $\left(x-\frac{y}{3}\right)(3x+10y)$. **79.** $(x+a)(x+b)$.

80. $\left(x^2+y^2+\frac{3xy}{2}\right)\left(x^2+y^2-\frac{3xy}{2}\right)$. **81.** $(x+6)(x+20)$.

82. $(x^2+x+1)(x^2-x+1)$.

83. $(a^2b^6+c^4)(ab^3+c^2)(ab^3-c^2)$.

84. $(x-1)(x^2+x+1)(x^3+2)$.

85. $(x^2-y^2+4xy)(x^2-y^2-4xy)$.

86. $4x(x+a)$.

87. $(x+11)(x+13)$.

88. $-a^2(x-3a)(x-3a)$.

89. $(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$.

90. $$x^4-[(a+b)^2+(a+c)^2]x^2+(a+b)^2(a+c)^2$$
$$=[x^2-(a+b)^2][x^2-(a+c)^2]$$
$$=(x+a+b)(x-a-b)(x+a+c)(x-a-c).$$

91. $(x^2+m)(ax^2+bx-am)$. **94.** $(2b)(2b)$.

92. $x^m y^{n+1}(ax^2y^2+bxy+c)$. **95.** $2(a-b)(1-ab)$.

93. $(x-ab+bc)(x+ab+ac)$. **96.** $(x-3bc)(x-ca-ab)$.

97. $11x(x^2+3y)(x^2+16y)$.

98. $[(a+b)x-(a-b)y][(a-b)x+(a+b)y]$.

99. $(x-y)(3x^2+3xy+4y^2)$.

100. $(1-abc)(1+2abc-a^2-b^2-c^2)$.

101. $(x+y+z)(x^2-yz)$.

102. $(a+b+c)(a-b-c)(c+a-b)(a+b-c)$.

103. $[(a+b)x+(a-b)y][(a-b)x+(a+b)y]$.

104. $(ax-by)(ay-bx)$. **107.** $2(x^5y^2-15)(x^5y^2-15)$.

105. $(ax+b)(bx+a)$. **108.** $(2ax-by)(2bx-ay)$.

106. $(x-c)(x+a+b)$. **109.** $(x+y)(b+c)(x+y-b-c)$.

110. $[x-(a-b)^2][x-(a+b)^2]$.

111. $(x-2y-6)(x+y+4)$.

112. $a^2x(a-x)(6ax+1)$.

113. $2(2+5ab)(2-5ab)$.

114. $(x+6y)(x-2y)$.

115. $(a+3b)(a-b)$.

116. $(x+8)(x+9)$.

117. $(x+\frac{1}{2})(x+\frac{1}{3})$.

118. $9(x^2+4)$.

119. $\left(\frac{x}{a}+\frac{a}{x}+1\right)\left(\frac{x}{a}+\frac{a}{x}-1\right)$.

120. $(x^3+27)(x^3-8)$
$=(x+3)(x^2-3x+9)(x-2)(x^2+2x+4)$.

121. $(x-11)(x-11)$.

122. $(6x+5)(10x-9)$.

123. $(x+1)(ax+b)$.

124. $(a+b+c)(a+b-c)(c+a-b)(c-a+b)$.

125. $(4x-y)(5x-y)$.

126. $\left(x^2+\frac{10x}{13}+1\right)\left(x^2-\frac{10x}{13}+1\right)$.

127. $(x+8)(x-7)$.

128. $x(x+y)(x^2-3xy+3y^2)$.

129. $(x^8+a^8)(x^4+a^4)(x^2+a^2)(x+a)(x-a)$.

130. $(7x+9y^2)(7x-9y^2)$.

131. $(x^2-5)(x^2-8)$.

132. $(x+\frac{1}{4})(x+\frac{1}{5})$.

133. $(x-11)(x+12)$.

134. $x(x-20)$.

135. $4x^2y^2(y+2x)(y-2x)$.

136. $3(1+2x)(1-2x)$.

137. $x^2(x-a^2)(x-a^2)$.

138. $(2x-3)(2x+1)$.

139. $(a+b)(a^2+b^2)$.

140. $(x^4+125)(x^4-125)(x^8+25^3)$.

141. $(x+a)(x-b)(x+c)$.

142. $-2a(x+y)$.

143. $(a+b+c)(a+b+c)$.

144. $(4xy+3)^2(4xy-3)^2$.

145. $(3x^2+3xy+2y^2)$
$(3x^2-3xy+2y^2)$.

146. $(x+b)(x+a+b)$.

147. $(x+1)(ax^2+bx+a)$.

148. $(a+b)(x-a)(x^2+ax+a^2)$.

149. $(a+b+c)(a-b-c)(c+a-b)(a+b-c)$.

150. $(x+y)[(a+b)x-3by]$.

151. $4(a-2)(a+2)(b+4)(b-4)$.

152. $(x-y-z)(x^2+y^2+z^2-yz+zx+xy)$.

153. $(a-x)(a+x)(b+x)$.

154. $(bcx-a^2)(bcx-1)$.

155. $(2x+3y)(2x-3y)^2$.

156. $(x+2a)(x-a-b)$.

157. $(ax+by)(by+c)$.

158. $5x^2(x+3y)(x+14y)$.

159. $(x-a)(x-b-c)$.

160. $(5x^{11}+4y^2)(16x^{11}-15y^2)$.

161. $\left(1+\frac{x}{2}+a+\frac{b}{2}\right)\left(1+\frac{x}{2}-a-\frac{b}{2}\right)$.

162. $(a-bx)(a+bx+cx^2)$.

163. $(x-a+3)(x-a-2)$.

164. $x(x+y)$.

165. $(x^2+yz)(x^4-4x^2yz+7y^2z^2)$.

166. $3(x^2+5y)(x^2+5y)$.

167. $(x-3)(x+1)$.

168. $(x^2-3x+1)(x^2+3x+1)$.

169. $(4x^2+9y^2)(2x+3y)(2x-3y)$.

170. $x^4(x^2+4a^2)(x+2a)(x-2a)$.

171. $(x^2+4xy+8y^2)(x^2-4xy+8y^2)$.

172. $(x^3-11)(x^3+10)$.

173. $2(4-3x)(2+7x)$.

174. $(2x-1)(x+1)$.

175. $x^2y(a+3y)(a^2-3ay+9y^2)$.

176. $x(x-1)(7x+103)$.
177. $(x-2)(x-4)$.
178. $(1+x)(1+14x)$.
179. $(x+6)(x+9)$.
180. $(x^2-4x+1)(x^2+4x+1)$.
181. $\left(x^2+\frac{5x}{19}+1\right)\left(x^2-\frac{5x}{19}+1\right)$.
182. $(x-a-2)(x+a+1)$.
183. $21(x-1)(3x+2)$.
184. $(2x-1)(x^2-2)$.
185. $(x+y)(x^2+y^2)$.
186. $3(x-7)(x+2)$.
187. $5x^2(3-2x)(5+x)$.
188. $(x-10)(x-20)$.
189. $4ab$.
190. $(x+5y)(x+6y)$.
191. $(3x-5y)(3x+14y)$.
192. $(x-a^2+b^2)(x+a^2-b^2)$.
193. $(a+b+c)(a+b+c)$.
194. $(ax+by)(bx-ay)$.
195. $(x+1)(2x^4+3x^2+4)$.
196. $(x+a)(x+2b)$.
197. $(a+b)(x+a)(x^2-ax+a^2)$.
198. $(x^2+4ax-4a^2)(x-2a)^2$.
199. $2(x+2a)(x+3b)$.
200. $(x^2-c^2-2b^2)(x^2+c^2-2a^2)$.
201. $3\left(1-\frac{a}{3}+x\right)\left(1-\frac{a}{3}-x\right)$.
202. $(ax+b)(bx-a)$.
203. $2b(x+y)$.
204. $(ax+by)(bx+ay)$.
205. $(a+b-c)(a-b+c)(a+b+c)(a-b-c)$.
206. $(x-a)^2(x+b)$.
207. $(x-2y+3z)(x-2y-3z)$.
208. $(4x+5b)(2x+3a)$.
209. $3(3a-3b-4x)(9a-9b+5x)$.
210. $(x-1)(x+1)(x^2+2x+2)$.
211. $2(2x^2-5y^2)(x^2+10y^2)$.

212. [illegible]
213. [illegible] $(x-2b-c)$.
214. [illegible]
215. [illegible]
216. [illegible]
217. $(x^2-5xy-y^2)(x^2-3xy-y^2)$.
218. $(x^2-15x-1)(x^2-11x-1)$.
219. [illegible]
220. $5ab(b-a)$.
221. $(x-12)(x-25)$.
222. [illegible]
223. $\frac{1}{3}(3x-10y)(3x-y)$.
224. $(x^2-15x-1)(x^2-15x-1)$.
225. $(x-1)(x-1)(x-5)(x-5)$.
226. $3y^2(x-y)(3x-2y)$.
227. $(x-a-b)(x-a-b)$.
228. [illegible]
229. $(x-y)(x-11y)$.
230. $(7xy-1)(4xy-1)$.
231. $(x-2)(x^2-4)$.
232. $4abxy$.
233. $(7x-y)(7x-6y)$.
234. $(1-ab)(1-18ab)$.
235. $(x-1)(ax-b)$.
236. $(x-2y)(x-6y)$.
237. $(x^2-y^2)(x-y)(x-y)(x^2-9y^2)(x-3y)(x-3y)$.
238. $(x^2-25)(x^2-81)$.
239. $(a^2-b^2)(a-2b)$.
240. $14(2x-3)(x-2)$.
241. $7(x^3-1)(x^3-2)$.
242. $(3x-5y)(7x-3y)$.
243. $(x-ca-ab)(x-bc+ca)$.
244. $(ab-c^2)(ac-b^2)$.
245. $(x^2-6y^2)(13x^2-40y^2)$.
246. $(ax-y)(x+ay)$.
247. $(x^2+mxy+y^2)(x^2-mxy-y^2)$.
248. $x(x-a)(x-b)$.
249. $(x+7)(x-2)(x+2)(x+3)$.
250. $[x^2+(1+m)y^2][x^2+(1-m)y^2]$.
251. $(9x^2-11y)(11x^2+12y)$.
252. $(x^2y-z^2)(x^2y-16z^2)$.
253. $(x-ab-4c)(x+ab-3c)$.
254. $(x^2+mxy-y^2)(x^2-mxy-y^2)$.
255. $(ax-by)(bx-ay)$.

256. $a(a-2b+3x)(a-2b-3x)$.

257. $(ax^2+bx+c)(ax^2-bx+c)$.

258. $(x-a)(x+a)(x-b)(x+b)$.

259. $(x+yz)(y+zx)$.

260. $(x+y-10z)(x+y+3z)$.

261. $(ax-c)(bx+c)$.

262. $(x+1)(x+2)(x+4)(x+5)$.

263. $3x(3x+5)$.

264. $(x^2-a^2-ab-b^2)(x^2-ab+a^2+b^2)$.

265. $(5x^2-3y^6)(9x^2+25y^6)$.

266. $(2a-b)(a^2+2bx)$.

267. $(x-a+b)(x+a+b)$.

268. $(x-b+a)(x-b-a)$.

269. $11(3x-4)(3x-5)$.

270. $ab(a^2-2ab-2b^2)$.

271. $(x^4y^2+3)(x^4y^2+12)$.

272. $12(4x^2-11)(4x^2+1)$.

273. $(x^2+y^2)(x+8y)(x-8y)$.

274. $(x-3y^2)(11x-21y^2)$.

275. $(x^2+4y)(x^2+9y)$.

276. $3(x+2y)(x+15y)$.

277. $(a-b)(a^2-2b^2)$.

278. $\frac{4}{15}(4a-b)(4b-a)$.

279. $x^2(6a+x)(4a-3x)$.

280. $(3x+2)(x^3-7)$.

281. $a^3(a^2+ab-b^2)(a^2-ab+b^2)$.

282. $(ax+y)(bx-y)$.

283. $(x-b)(x+b)(x-a^2)$.

284. $(a^4+b^4)(a^2+b^2)^2(a+b)^2(a-b)^2$.

285. $(x-a)(x+a)(x+b)$.

286. $(x+y)(x-y)^3$.

287. $(2+ax)(a-2x)$.

288. $5(x^4-25)+4x(x^2-5)$
$= (x^2-5)(5x^2+4x+25)$.

289. $(a+bx)(a-bx+cx^2)$.

290. $x^2(6x-7)(6x+5)$.

291. $2a(a^2+3b^2)$.

Ex. 9. Page 31.

1. $2(x+1)$.
2. $9(a+b)$.
3. $3(x+1)$.
4. $4(x^2+y^2)$.
5. $a-x$.
6. $ax(a-x)^2$.
7. a^2-b^2.
8. $x-y$.
9. $3(a^2c+2b^2)$.
10. $2a+3b$.
11. $b-a$.
12. $2a+5$.
13. $4a+3b$.
14. $5x-1$.
15. $x^5(3a^3-4ax^2y^2-6x^3y)$.
16. $x+y$.
17. $(a+b)^2$.
18. $2x-3y$.
19. $a+b$.
20. $x+5$.
21. $x-7$.
22. $x-10$.
23. $x-12$.
24. $x+1$.
25. $3x-2$.
26. $3x-2a$.
27. $2x+3$.
28. $x-1$.
29. $x(x+3)$.
30. $2(7-3x)$.
31. $x+1$.
32. x^2+7.
33. $x-y$.
34. $a+5$.
35. $x-y$.
36. $a-b$.
37. $x-a$.
38. $(x-y)^2$.
39. $5x^2-1$.
40. x^2-4x+4.
41. x^2-4y^2.
42. $a(a^2-2a+1)$.
43. $9x^2(x-1)$.
44. x^2-y^2.
45. a^3+a^2-5a+3.
46. $x+3$.
47. a^2+b^2.
48. $a(2a^2+2ab-b^2)$.
49. $7x^2+8x+1$.
50. $x^4-2x^3+3x^2$ $-2x+1$.
51. x^2-3x+1.
52. $x+1$.
53. $a(2a-3x)$.
54. $(x+1)^3$.
55. $2x^3-4x^2+x-1$.
56. $x-2a$.
57. $x+a$.
58. $x^2-(a+b)x+ab$.
59. $2x-9$.
60. x^2-1.
61. $x^2-(a-b)x+b^2$.
62. x^3+x^2+x+1.
63. $x+1$.
64. $x+a$.
65. $a-b$.
66. $x-2a$.
67. $a+b$.
68. $3x-y$.

Ex. 10. Page 34.

1. $100a^5b^4c^6$.
2. $ab(a+b)$.
3. $12ab(b-1)$.
4. $90b(a^2-b^2)$.
5. $(a+b)(c^2-d^2)$.
6. $(a+b)(a-b)^2$.
7. $12(a^2-x^2)$.
8. $ab(a^2x^2-b^2y^2)$.
9. $b^2(b+ax)$.

10. $(x+1)(x+3)(x-4)$.

11. x^3+3x^2-x-3.

12. $12x^3-8x^2-27x+18$.

13. $x(2x+1)(3x-1)(4x+3)$.

14. $(x+2)(x+4)(x^2+3x+1)$.

15. $36x^3-3ax^2-50a^2x+24a^3$, or $(4x-3a)(3x-2a)(3x+4a)$.

16. $(a^3-9a^2+23a-15)(a-7)$.

17. $(a^2-b^2)(a^2-4b^2)$.

18. $60x^6y^2+190x^5y^3+116x^4y^4+64x^3y^5+48x^2y^6-30xy^7$.

19. a^6-b^6.

20. $mnpq$.

21. $36x^3y^3z^3$.

22. x^2-y^2.

23. $(x+y)(x-y)^2$.

24. $6a(a^2+3a+2)$.

25. $a(a^2-4b^2)$.

26. $a(4a^2-1)$.

27. $2a(a^2-b^2)$.

28. x^3-1.

29. $(x+1)(x+2)(x+3)$.

30. $(x^3-19x-30)(x^2+5x+10)$.

31. $(x+1)(x+2)(x+3)(x-1)$.

32. $(4x^2-9)(9x^2-4)$.

33. $(a-1)(a-2)(a-3)(a-4)$.

34. x^8-1.

35. $x^{12}-1$.

36. $(x-a)(x-b)(x-c)$.

37. $(x+c)(2x-3b)(x^2+ax-b^2)$.

Ex. 11. Page 35.

1. $\frac{3n^3}{2q}$.

2. $\frac{2a}{3b}$.

3. $\frac{3b^2}{5ac}$.

4. $\frac{6a^2}{5x^2}$.

5. $\frac{4a}{7b^2c^2}$.

6. $\frac{4ab}{9x}$.

7. $\frac{7b^2-4ac}{6c^2-7ab}$.

8. $\frac{8a^2b-x^3}{9a^3-b^2x}$.

9. $\frac{2x^4-3a^2b^2}{3bx^3-4a^4}$.

10. $\frac{a+b}{a(1-3c^2)}$.

11. $\frac{b^2-3ac}{c^2-4ab}$.

12. $\frac{2x^2-y}{3x^2-y^2}$.

13. $\frac{2z^2-3y^2}{3z^2-2y^2}$.

14. $\frac{3c^2-a^2}{2a^2+c^2}$.

15. $\frac{3a^2-7c}{4a^2+9b^2}$.

16. $\dfrac{5a^2-3b}{2(2b^2-3a)}$.

17. $\dfrac{1}{2x+1}$.

18. $\dfrac{1}{3x-1}$.

19. $\dfrac{1}{8x^3-1}$.

20. $\dfrac{x^2}{x-3}$.

21. $\dfrac{xy}{x+1}$.

22. $\dfrac{x^2-y}{x}$.

23. $\dfrac{x(3x^2+2)}{2y^2}$.

24. $\dfrac{x^2y^2}{x^2z^2+y^4}$.

25. $\dfrac{3x}{4(a-3x)}$.

26. $\dfrac{x^2}{x+y}$.

27. $\dfrac{x^2}{x+2}$.

28. $\dfrac{4x}{3x+2y}$.

29. $\dfrac{3b^2}{a^2-2c}$.

30. $\dfrac{1}{5x(7x^2+3)}$.

31. $\dfrac{3a}{5a-9b}$.

32. $\dfrac{(a-b)(a^2+b^2)}{a+b}$.

33. $\dfrac{x+z}{(1-y)^2}$.

34. $\dfrac{ax}{(a+b)^2}$.

35. $\dfrac{4a^3x^2}{3b(5a^2+4b)}$.

36. $\dfrac{x^2-ax+b^2}{x^2+ax-b^2}$.

37. $\dfrac{x+c}{a+b-x}$.

38. $\dfrac{x-a}{x-b+c}$.

39. $\dfrac{x-b}{bc-a(c+x)}$.

40. $\dfrac{bc}{(a+b+c)(b+c-a)}$.

41. $\dfrac{x+y+z}{x-y-z}$.

42. $\dfrac{x+2}{x+5}$.

43. $\dfrac{x+7}{x-5}$.

44. $\dfrac{x-2}{x-3}$.

45. $\dfrac{x-2}{x+1}$.

46. $\dfrac{x-4}{x-5}$.

47. $\dfrac{x+4}{x+6}$.

48. $\dfrac{a-7b}{a+b}$.

49. $\dfrac{a+b}{a-b}$.

50. $\dfrac{x+3}{x-3}$.

51. $\dfrac{2x-3}{7x-3}$.

52. $\dfrac{4a-3b}{5a+2b}$.

53. $\dfrac{2x-1}{x-1}$.

54. $\dfrac{3x-4y}{5x-3y}$.

55. $\dfrac{4y+3}{3(y+2)}$.

56. $\dfrac{(2a+1)x}{a+1}$.

57. $\dfrac{(x-1)^2}{x^2-3x+1}$.

58. $\dfrac{a^2+b^2}{a}$.

59. $\dfrac{1}{b-2x}$.

60. $\dfrac{7(x^2+xy+y^2)}{5}$.

Ex. 12. Page 38.

1. $\frac{a+2m^2}{m}$.
2. $\frac{3x-a}{x-a}$.
3. $\frac{4b}{3a+2b}$.
4. $\frac{3a^2-2b^2}{6ab}$.
5. $\frac{5ay-2bx}{x^2y}$.
6. $\frac{x^2-2y^2}{10xy}$.
7. $\frac{5n+ax}{5a^2}$.
8. 2.
9. $\frac{b^2-a^2}{ab}$.
10. $\frac{3a+b}{2b}$.
11. $\frac{4a+3b}{12a}$.
12. $\frac{21a-16b}{6}$.
13. $\frac{133a}{36}$.
14. $\frac{2a+3b}{b}$.
15. $\frac{x+y}{3}$.
16. $\frac{a-b}{2}$.
17. $-\frac{x+5y}{15}$.
18. $\frac{a}{12}$.
19. 0.
20. $\frac{4a}{45}$.
21. $\frac{3b+5c-5a}{36}$.
22. $\frac{11a-8b+3c}{84}$.
23. 0.
24. $a+b$.
25. $\frac{5a+286b}{30b}$.
26. $\frac{ab+ac+bc}{abc}$.
27. $\frac{a(np+mp+mn)}{mnp}$.
28. $\frac{az-by-cx}{xyz}$.
29. $\frac{2ab+3bx-4ax}{abx^2}$.
30. $\frac{7}{60b}$.
31. $\frac{a}{6x}$
32. $\frac{a(3df-5cf+4cd)}{bcdf}$.
33. $\frac{2(9adg+60ad-5cg)}{45bdg}$.
34. $\frac{a}{18x}$.
35. $\frac{ab+ac+bc}{abc}$.
36. $\frac{a^2+b^2+c^2}{abc}$.
37. $\frac{a^2-x^2}{2a^2x^2}$.
38. $\frac{2a}{a^2-b^2}$.
39. $\frac{a^2+2ab-b^2}{a^2-b^2}$.

78. $\left(x-\frac{y}{3}\right)(3x+10y)$.

79. $(x+a)(x+b)$.

80. $\left(x^2+y^2+\frac{3xy}{2}\right)\left(x^2+y^2-\frac{3xy}{2}\right)$.

81. $(x+6)(x+20)$.

82. $(x^2+x+1)(x^2-x+1)$.

83. $(a^2b^6+c^4)(ab^3+c^2)(ab^3-c^2)$.

84. $(x-1)(x^2+x+1)(x^3+2)$.

85. $(x^2-y^2+4xy)(x^2-y^2-4xy)$.

86. $4x(x+a)$.

87. $(x+11)(x+13)$.

88. $-a^2(x-3a)(x-3a)$.

89. $(a-b)(a+b)(a^2+ab+b^2)(a^2-ab+b^2)$.

90. $$\begin{aligned} &x^4-[(a+b)^2+(a+c)^2]x^2+(a+b)^2(a+c)^2 \\ &=[x^2-(a+b)^2][x^2-(a+c)^2] \\ &=(x+a+b)(x-a-b)(x+a+c)(x-a-c). \end{aligned}$$

91. $(x^2+m)(ax^2+bx-am)$.

92. $x^my^{n+1}(ax^2y^2+bxy+c)$.

93. $(x-ab+bc)(x+ab+ac)$.

94. $(2b)(2b)$.

95. $2(a-b)(1-ab)$.

96. $(x-3bc)(x-ca-ab)$.

97. $11x(x^2+3y)(x^2+16y)$.

98. $[(a+b)x-(a-b)y][(a-b)x+(a+b)y]$.

99. $(x-y)(3x^2+3xy+4y^2)$.

100. $(1-abc)(1+2abc-a^2-b^2-c^2)$.

101. $(x+y+z)(x^2-yz)$.

102. $(a+b+c)(a-b-c)(c+a-b)(a+b-c)$.

103. $[(a+b)x+(a-b)y][(a-b)x+(a+b)y]$.

104. $(ax-by)(ay-bx)$.

105. $(ax+b)(bx+a)$.

106. $(x-c)(x+a+b)$.

107. $2(x^5y^2-15)(x^5y^2-15)$.

108. $(2ax-by)(2bx-ay)$.

109. $(x+y)(b+c)(x+y-b-c)$.

110. $[x-(a-b)^2][x-(a+b)^2]$.

111. $(x-2y-6)(x+y+4)$.

112. $a^2x(a-x)(6ax+1)$.

113. $2(2+5ab)(2-5ab)$.

114. $(x+6y)(x-2y)$.

115. $(a+3b)(a-b)$.

116. $(x+8)(x+9)$.

117. $(x+\frac{1}{2})(x+\frac{1}{3})$.

118. $9(x^2+4)$.

119. $\left(\frac{x}{a}+\frac{a}{x}+1\right)\left(\frac{x}{a}+\frac{a}{x}-1\right)$.

120. $(x^3+27)(x^3-8)$
$=(x+3)(x^2-3x+9)(x-2)(x^2+2x+4)$.

121. $(x-11)(x-11)$.

122. $(6x+5)(10x-9)$.

123. $(x+1)(ax+b)$.

124. $(a+b+c)(a+b-c)(c+a-b)(c-a+b)$.

125. $(4x-y)(5x-y)$.

126. $\left(x^2+\frac{10x}{13}+1\right)\left(x^2-\frac{10x}{13}+1\right)$.

127. $(x+8)(x-7)$.

128. $x(x+y)(x^2-3xy+3y^2)$.

129. $(x^8+a^8)(x^4+a^4)(x^2+a^2)(x+a)(x-a)$.

130. $(7x+9y^2)(7x-9y^2)$.

131. $(x^2-5)(x^2-8)$.

132. $(x+\frac{1}{4})(x+\frac{1}{5})$.

133. $(x-11)(x+12)$.

134. $x(x-20)$.

135. $4x^2y^2(y+2x)(y-2x)$.

136. $3(1+2x)(1-2x)$.

137. $x^2(x-a^2)(x-a^2)$.

138. $(2x-3)(2x+1)$.

139. $(a+b)(a^2+b^2)$.

140. $(x^4+125)(x^4-125)(x^8+25^3)$.

141. $(x+a)(x-b)(x+c)$.

142. $-2a(x+y)$.

143. $(a+b+c)(a+b+c)$.

144. $(4xy+3)^2(4xy-3)^2$.

145. $(3x^2+3xy+2y^2)$
$(3x^2-3xy+2y^2)$.

146. $(x+b)(x+a+b)$.

147. $(x+1)(ax^2+bx+a)$.

148. $(a+b)(x-a)(x^2+ax+a^2)$.

149. $(a+b+c)(a-b-c)(c+a-b)(a+b-c)$.

150. $(x+y)[(a+b)x-3by]$.

151. $4(a-2)(a+2)(b+4)(b-4)$.

152. $(x-y-z)(x^2+y^2+z^2-yz+zx+xy)$.

153. $(a-x)(a+x)(b+x)$.

154. $(bcx-a^2)(bcx-1)$.

155. $(2x+3y)(2x-3y)^2$.

156. $(x+2a)(x-a-b)$.

157. $(ax+by)(by+c)$.

158. $5x^2(x+3y)(x+14y)$.

159. $(x-a)(x-b-c)$.

160. $(5x^{11}+4y^2)(16x^{11}-15y^2)$.

161. $\left(1+\frac{x}{2}+a+\frac{b}{2}\right)\left(1+\frac{x}{2}-a-\frac{b}{2}\right)$.

162. $(a-bx)(a+bx+cx^2)$.

163. $(x-a+3)(x-a-2)$.

164. $x(x+y)$.

165. $(x^2+yz)(x^4-4x^2yz+7y^2z^2)$.

166. $3(x^2+5y)(x^2+5y)$.

167. $(x-3)(x+1)$.

168. $(x^2-3x+1)(x^2+3x+1)$.

169. $(4x^2+9y^2)(2x+3y)(2x-3y)$.

170. $x^4(x^2+4a^2)(x+2a)(x-2a)$.

171. $(x^2+4xy+8y^2)(x^2-4xy+8y^2)$.

172. $(x^3-11)(x^3+10)$.

173. $2(4-3x)(2+7x)$.

174. $(2x-1)(x+1)$.

175. $x^2y(a+3y)(a^2-3ay+9y^2)$.

176. $x(x-1)(7x+103)$.

177. $(x-2)(x-4)$.

178. $(1+x)(1+14x)$.

179. $(x+6)(x+9)$.

180. $(x^2-4x+1)(x^2+4x+1)$.

181. $\left(x^2+\frac{5x}{19}+1\right)\left(x^2-\frac{5x}{19}+1\right)$.

182. $(x-a-2)(x+a+1)$.

183. $21(x-1)(3x+2)$.

184. $(2x-1)(x^2-2)$.

185. $(x+y)(x^2+y^2)$.

186. $3(x-7)(x+2)$.

187. $5x^2(3-2x)(5+x)$.

188. $(x-10)(x-20)$.

189. $4ab$.

190. $(x+5y)(x+6y)$.

191. $(3x-5y)(3x+14y)$.

192. $(x-a^2+b^2)(x+a^2-b^2)$.

193. $(a+b+c)(a+b+c)$.

194. $(ax+by)(bx-ay)$.

195. $(x+1)(2x^4+3x^2+4)$.

196. $(x+a)(x+2b)$.

197. $(a+b)(x+a)(x^2-ax+a^2)$.

198. $(x^2+4ax-4a^2)(x-2a)^2$.

199. $2(x+2a)(x+3b)$.

200. $(x^2-c^2-2b^2)(x^2+c^2-2a^2)$.

201. $3\left(1-\frac{a}{3}+x\right)\left(1-\frac{a}{3}-x\right)$.

202. $(ax+b)(bx-a)$.

203. $2b(x+y)$.

204. $(ax+by)(bx+ay)$.

205. $(a+b-c)(a-b+c)(a+b+c)(a-b-c)$.

206. $(x-a)^2(x+b)$.

207. $(x-2y+3z)(x-2y-3z)$.

208. $(4x+5b)(2x+3a)$.

209. $3(3a-3b-4x)(9a-9b+5x)$.

210. $(x-1)(x+1)(x^2+2x+2)$.

211. $2(2x^2-5y^2)(x^2+10y^2)$.

212. $(x+2a)(x+3b)$.
213. $(a+2b+c)(2b+c-a)(c+a-2b)(a+2b-c)$.
214. $3x^2y^2z^2(x^2y-2y^2z+4xz^2)$.
215. $(x-2a)(x-b)$.
216. $7(2x+1)(2x-5)$.
217. $(x^2-3xy-y^2)(x^2+3xy-y^2)$.
218. $(x^2-13x-1)(x^2+13x-1)$.
219. $(a+b)^2(a-b)^2$.
220. $3ab(b-a)$.
221. $(x-12)(x+25)$.
222. $5(1+7x^2y^2)(1-7x^2y^2)$.
223. $\frac{5}{6}(3x-10y)(3x+y)$.
224. $(x^2+13x+1)(x^2-13x+1)$.
225. $(x+1)(x-1)(x+3)(x-3)$.
226. $3y^2(x-y)(3x+2y)$.
227. $(x+a+b)(x+a-b)$.
228. $5(x-1)(x+1)$.
229. $(x-y)(x+10y)$.
230. $(7xy-1)(4xy+1)$.
231. $(x+2)(x^2+4)$.
232. $4abxy$.
233. $(7x+y)(7x+6y)$.
234. $(1+ab)(1+18ab)$.
235. $(x+1)(ax-b)$.
236. $(x-2y)(x-6y)$.
237. $(x^2+y^2)(x+y)(x-y)(x^2+9y^2)(x+3y)(x-3y)$.
238. $(x^3+20)(x^3+31)$.
239. $(a^2+b^2)(a-2b)$.
240. $14(2x+3)(x+2)$.
241. $7(x^5-1)(x^5-2)$.
242. $(3x+5y)(7x-3y)$.
243. $(x-ca-ab)(x+bc+ca)$.
244. $(ab-c^2)(ac-b^2)$.
245. $(x^2-6y^2)(13x^2-40y^2)$.
246. $(ax+y)(x+ay)$.
247. $(x^2+mxy+y^2)(x^2-mxy+y^2)$.
248. $x(x-a)(x-b)$.
249. $(x+7)(x-2)(x+2)(x+3)$.
250. $[x^2+(1+m)y^2][x^2+(1-m)y^2]$.
251. $(9x^2-11y)(11x^2+12y)$.
252. $(x^2y-z^2)(x^2y-16z^2)$.
253. $(x-ab-4c)(x+ab-3c)$.
254. $(x^2+mxy-y^2)(x^2-mxy-y^2)$.
255. $(ax+by)(bx-ay)$.

256. $a(a-2b+3x)(a-2b-3x)$.

257. $(ax^2+bx+c)(ax^2-bx+c)$.

258. $(x-a)(x+a)(x-b)(x+b)$.

259. $(x+yz)(y+zx)$.

260. $(x+y-10z)(x+y+3z)$.

261. $(ax-c)(bx+c)$.

262. $(x+1)(x+2)(x+4)(x+5)$.

263. $3x(3x+5)$.

264. $(x^2-a^2-ab-b^2)(x^2-ab+a^2+b^2)$.

265. $(5x^2-3y^6)(9x^2+25y^6)$.

266. $(2a-b)(a^2+2bx)$.

267. $(x-a+b)(x+a+b)$.

268. $(x-b+a)(x-b-a)$.

269. $11(3x-4)(3x-5)$.

270. $ab(a^2-2ab-2b^2)$.

271. $(x^4y^2+3)(x^4y^2+12)$.

272. $12(4x^2-11)(4x^2+1)$.

273. $(x^2+y^2)(x+8y)(x-8y)$.

274. $(x-3y^2)(11x-21y^2)$.

275. $(x^2+4y)(x^2+9y)$.

276. $3(x+2y)(x+15y)$.

277. $(a-b)(a^2-2b^2)$.

278. $\frac{4}{15}(4a-b)(4b-a)$.

279. $x^2(6a+x)(4a-3x)$.

280. $(3x+2)(x^3-7)$.

281. $a^3(a^2+ab-b^2)(a^2-ab+b^2)$.

282. $(ax+y)(bx-y)$.

283. $(x-b)(x+b)(x-a^2)$.

284. $(a^4+b^4)(a^2+b^2)^2(a+b)^2(a-b)^2$.

285. $(x-a)(x+a)(x+b)$.

286. $(x+y)(x-y)^3$.

287. $(2+ax)(a-2x)$.

288. $5(x^4-25)+4x(x^2-5)$
$= (x^2-5)(5x^2+4x+25)$.

289. $(a+bx)(a-bx+cx^2)$.

290. $x^2(6x-7)(6x+5)$.

291. $2a(a^2+3b^2)$.

Ex. 9. Page 31.

1. $2(x+1)$.
2. $9(a+b)$.
3. $3(x+1)$.
4. $4(x^2+y^2)$.
5. $a-x$.
6. $ax(a-x)^2$.
7. a^2-b^2.
8. $x-y$.
9. $3(a^2c+2b^2)$.
10. $2a+3b$.
11. $b-a$.
12. $2a+5$.
13. $4a+3b$.
14. $5x-1$.
15. $x^5(3a^3-4ax^2y^2-6x^3y)$.
16. $x+y$.
17. $(a+b)^2$.
18. $2x-3y$.
19. $a+b$.
20. $x+5$.
21. $x-7$.
22. $x-10$.
23. $x-12$.
24. $x+1$.
25. $3x-2$.
26. $3x-2a$.
27. $2x+3$.
28. $x-1$.
29. $x(x+3)$.
30. $2(7-3x)$.
31. $x+1$.
32. x^2+7.
33. $x-y$.
34. $a+5$.
35. $x-y$.
36. $a-b$.
37. $x-a$.
38. $(x-y)^2$.
39. $5x^2-1$.
40. x^2-4x+4.
41. x^2-4y^2.
42. $a(a^2-2a+1)$.
43. $9x^2(x-1)$.
44. x^2-y^2.
45. a^3+a^2-5a+3.
46. $x+3$.
47. a^2+b^2.
48. $a(2a^2+2ab-b^2)$.
49. $7x^2+8x+1$.
50. $x^4-2x^3+3x^2-2x+1$.
51. x^2-3x+1.
52. $x+1$.
53. $a(2a-3x)$.
54. $(x+1)^3$.
55. $2x^3-4x^2+x-1$.
56. $x-2a$.
57. $x+a$.
58. $x^2-(a+b)x+ab$.
59. $2x-9$.
60. x^2-1.
61. $x^2-(a-b)x+b^2$.
62. x^3+x^2+x+1.
63. $x+1$.
64. $x+a$.
65. $a-b$.
66. $x-2a$.
67. $a+b$.
68. $3x-y$.

Ex. 10. Page 34.

1. $100a^5b^4c^6$.
2. $ab(a+b)$.

$b(b-1)$.

4. $90b(a^2-b^2)$.
5. $(a+b)(c^2-d^2)$.
6. $(a+b)(a-b)^2$.
7. $12(a^2-x^2)$.
8. $ab(a^2x^2-b^2y^2)$.
9. $b^2(b+ax)$.

10. $(x+1)(x+3)(x-4)$.

11. x^3+3x^2-x-3.

12. $12x^3-8x^2-27x+18$.

13. $x(2x+1)(3x-1)(4x+3)$.

14. $(x+2)(x+4)(x^2+3x+1)$.

15. $36x^3-3ax^2-50a^2x+24a^3$, or $(4x-3a)(3x-2a)(3x+4a)$.

16. $(a^3-9a^2+23a-15)(a-7)$.

17. $(a^2-b^2)(a^2-4b^2)$.

18. $60x^6y^2+190x^5y^3+116x^4y^4+64x^3y^5+48x^2y^6-30xy^7$.

19. a^6-b^6.

20. $mnpq$.

21. $36x^3y^3z^3$.

22. x^2-y^2.

23. $(x+y)(x-y)^2$.

24. $6a(a^2+3a+2)$.

25. $a(a^2-4b^2)$.

26. $a(4a^2-1)$.

27. $2a(a^2-b^2)$.

28. x^3-1.

29. $(x+1)(x+2)(x+3)$.

30. $(x^3-19x-30)(x^2+5x+10)$.

31. $(x+1)(x+2)(x+3)(x-1)$.

32. $(4x^2-9)(9x^2-4)$.

33. $(a-1)(a-2)(a-3)(a-4)$.

34. x^8-1.

35. $x^{12}-1$.

36. $(x-a)(x-b)(x-c)$.

37. $(x+c)(2x-3b)(x^2+ax-b^2)$.

Ex. 11. Page 35.

1. $\dfrac{3n^3}{2q}$.

2. $\dfrac{2a}{3b}$.

3. $\dfrac{3b^2}{5ac}$.

4. $\dfrac{6a^2}{5x^2}$.

5. $\dfrac{4a}{7b^2c^2}$.

6. $\dfrac{4ab}{9x}$.

7. $\dfrac{7b^2-4ac}{6c^2-7ab}$.

8. $\dfrac{8a^2b-x^3}{9a^3-b^2x}$.

9. $\dfrac{2x^4-3a^2b^2}{3bx^3-4a^4}$.

10. $\dfrac{a+b}{a(1-3c^2)}$.

11. $\dfrac{b^2-3ac}{c^2-4ab}$.

12. $\dfrac{2x^2-y}{3x^2-y^2}$.

13. $\dfrac{2z^2-3y^2}{3z^2-2y^2}$.

14. $\dfrac{3c^2-a^2}{2a^2+c^2}$.

15. $\dfrac{3a^2-7c}{4a^2+9b^2}$.

16. $\frac{5a^2-3b}{2(2b^2-3a)}$.

17. $\frac{1}{2x+1}$.

18. $\frac{1}{3x-1}$.

19. $\frac{1}{8x^3-1}$.

20. $\frac{x^2}{x-3}$.

21. $\frac{xy}{x+1}$.

22. $\frac{x^2-y}{x}$.

23. $\frac{x(3x^2+2)}{2y^2}$.

24. $\frac{x^2y^2}{x^2z^2+y^4}$.

25. $\frac{3x}{4(a-3x)}$.

26. $\frac{x^2}{x+y}$.

27. $\frac{x^2}{x+2}$.

28. $\frac{4x}{3x+2y}$.

29. $\frac{3b^2}{a^2-2c}$.

30. $\frac{1}{5x(7x^2+3)}$.

31. $\frac{3a}{5a-9b}$.

32. $\frac{(a-b)(a^2-b^2)}{a+b}$.

33. $\frac{x+z}{(1-z)^2}$.

34. $\frac{ax}{(a+b)^2}$.

35. $\frac{4a^3x^2}{3b(5a^2-4b)}$

36. $\frac{x^2-ax+b^2}{x^2+ax-b^2}$.

37. $\frac{x+c}{a+b-x}$.

38. $\frac{x-a}{x-b+c}$

39. $\frac{x-b}{bc-a(c+x)}$

40. $\frac{bc}{(a+b+c)(b+c-a)}$

41. $\frac{x+y+z}{x-y-z}$.

42. $\frac{x+2}{x+5}$.

43. $\frac{x+7}{x-5}$.

44. $\frac{x-2}{x-3}$

45. $\frac{x-2}{x+1}$.

46. $\frac{x-4}{x-5}$.

47. $\frac{x+4}{x+6}$.

48. $\frac{a-7b}{a+b}$.

49. $\frac{a+b}{a-b}$.

50. $\frac{x+3}{x-3}$.

51. $\frac{2x-3}{7x-3}$.

52. $\frac{4a-3b}{5a+2b}$.

53. $\frac{2x-1}{x-1}$.

54. $\frac{3x-4y}{5x-3y}$.

55. $\frac{4y+3}{3(y+2)}$.

56. $\frac{(2a+1)x}{a+1}$.

57. $\frac{(x-1)^2}{x^2-3x+1}$.

58. $\frac{a^2+b^2}{a}$.

59. $\frac{1}{b-2x}$.

60. $\frac{7(x^2+xy+y^2)}{5}$.

Ex. 12. Page 38.

1. $\frac{a+2m^2}{m}$.
2. $\frac{3x-a}{x-a}$.
3. $\frac{4b}{3a+2b}$.
4. $\frac{3a^2-2b^2}{6ab}$.
5. $\frac{5ay-2bx}{x^2y}$.
6. $\frac{x^2-2y^2}{10xy}$.
7. $\frac{5n+ax}{5a^2}$.
8. 2.
9. $\frac{b^2-a^2}{ab}$.
10. $\frac{3a+b}{2b}$.
11. $\frac{4a+3b}{12a}$.
12. $\frac{21a-16b}{6}$.
13. $\frac{133a}{36}$.
14. $\frac{2a+3b}{b}$.
15. $\frac{x+y}{3}$.
16. $\frac{a-b}{2}$.
17. $-\frac{x+5y}{15}$.
18. $\frac{a}{12}$.
19. 0.
20. $\frac{4a}{45}$.
21. $\frac{3b+5c-5a}{36}$.
22. $\frac{11a-8b+3c}{84}$.
23. 0.
24. $a+b$.
25. $\frac{5a+286b}{30b}$.
26. $\frac{ab+ac+bc}{abc}$.
27. $\frac{a(np+mp+mn)}{mnp}$.
28. $\frac{az-by-cx}{xyz}$.
29. $\frac{2ab+3bx-4ax}{abx^2}$.
30. $\frac{7}{60b}$.
31. $\frac{a}{6x}$
32. $\frac{a(3df-5cf+4cd)}{bcdf}$.
33. $\frac{2(9adg+60ad-5cg)}{45bdg}$.
34. $\frac{a}{18x}$.
35. $\frac{ab+ac+bc}{abc}$.
36. $\frac{a^2+b^2+c^2}{abc}$.
37. $\frac{a^2-x^2}{2a^2x^2}$.
38. $\frac{2a}{a^2-b^2}$.
39. $\frac{a^2+2ab-b^2}{a^2-b^2}$.

40. $\frac{2bc}{a^2-b^2}$.

41. $\frac{1}{x-y}$.

42. $\frac{12a}{1-9a^2}$.

43. $\frac{a+x}{ax}$.

44. $\frac{a+b}{2(a-b)}$.

45. $\frac{6x}{2a-3x}$.

46. $\frac{3a^2-2ab+3b^2}{2(a^2-b^2)}$.

47. $\frac{3x+2}{x-1}$.

48. $\frac{6ax}{a-x}$.

49. $\frac{2xa^n}{a^{2n}-1}$.

50. $\frac{4a}{a+x}$.

51. $\frac{b(a+b)}{x^2-b^2}$.

52. $\frac{b}{a-b}$.

53. $\frac{2x-3}{x(4x^2-1)}$.

54. $\frac{2a+b}{2a-b}$.

55. $\frac{2}{x(1-4x^2)}$.

56. $\frac{16}{(x-2)(x+2)^2}$.

57. $\frac{a^4+6a^2x^2+x^4}{a^4-x^4}$.

58. $\frac{2}{(x+1)(x+2)(x+3)}$.

59. $\frac{x(5x-7)}{(x^2-1)(x-2)}$.

60. $\frac{2x-3}{(x^2-1)(2x+3)}$.

61. $\frac{8x^4-x^3-24x^2+5x-4}{(x^2-1)(x+3)(x-2)}$.

62. $\frac{3a^4+4a^3x+6a^2x^2-4ax^3+3x^4}{a^4-x^4}$.

63. $\frac{2x^2+21x+13}{(1-x^2)(1-4x^2)}$.

64. $\frac{2}{x^4-y^4}$.

65. $\frac{18(2a+15)}{16a^4-81}$.

66. $\frac{x+c}{(x-a)(b-x)}$.

67. 0.

68. 0.

69. $\frac{x^2-4ax-a^2}{(x^2-a^2)^2}$.

70. 0.

71. 0.

72. -1.

73. $\frac{1}{abc}$.

74. 0.

75. $\frac{2x^2-9x+44}{x^3+64}$.

76. $\frac{2a}{x^2-a^2}$.

77. $\frac{x^3+1}{x^4-3x^3+3x^2-x}$.

78. 1.

79. $\frac{x(x-2)}{x^3+1}$.

80. $\frac{2x^2}{x^3+y^3}$.

81. $\frac{2y^2}{x^3-y^3}$.

82. $\frac{4-3x+2x^3-3x^4}{2(1+x^2)(1+x^3)}$.

83. $\frac{2(x+1)}{x^2+x+1}$.

84. $\frac{3x^2+9}{x^3-27}$.

85. $\dfrac{4a^3}{a^8+a^4+1}$. **86.** 0. **91.** $\dfrac{4}{x+2}$.

87. $-\dfrac{2(3x+4)}{(x-1)^2}$. **92.** $\dfrac{5a}{x-5a}$.

88. $-\dfrac{2x^4+10x^3-9x^2-16x+4}{(4-x^2)^2}$. **93.** $\dfrac{a(x-9a)}{(x-6a)(x-4a)}$.

89. $\dfrac{9x^2+1}{(2x-1)^3}$. **94.** $\dfrac{12+25x-x^2}{(3+x^2)(3+x)^2}$.

90. $\dfrac{7a^2-4ax-2x^2}{2(a-2x)^3}$. **95.** $\dfrac{84-186x+93x^2-6x^3}{4(3-2x)^2(3+2x)}$.

96. $\dfrac{(z^4-2z^2-3)(4z^2-3)-(z^2-4z+1)(5z^4+5z^2-9)}{(4z^2-3)(3z^2+2)(5z^4+5z^2-9)}$.

97. $\dfrac{4x^3+40x^2+70x-22}{x^4+5x^3-13x^2-77x-60}$.

98. $\dfrac{11-x^2}{(x-2)(x-5)(x-7)}$. **99.** $\dfrac{3a^2-41ax+74x^2}{3(2a-5x)^3}$.

100. 0.

Ex. 13. Page 43.

1. $\dfrac{18p^2q^9r^8}{35m^2x^{11}y^{13}}$. 7. $\dfrac{3a^9}{5bc}$. 13. $\dfrac{8a^6b^9x^{12}}{27y^{15}}$.

2. $\dfrac{8x^3}{3yz^2}$. 8. $\dfrac{2m^{12}n}{a^2b^{25}}$. 14. $\dfrac{a^{10}y^{5p}x^{5n-15}}{32b^{5x-5}z^{20}}$.

3. $\dfrac{4d}{9acq}$. 9. $\dfrac{a^2}{b^2}$. 15. $\dfrac{1}{9a^5x^2}$.

4. 1. 10. $\dfrac{8a^3}{125b^3}$. 16. $\dfrac{9a^{10}c^6x}{2b^{11}}$.

5. $\dfrac{ab^2}{y^3}$. 11. $\dfrac{625x^4y^4}{2401a^4b^4}$. 17. $\dfrac{16b^3}{27d^3}$.

6. $\dfrac{3c^2}{8}$. 12. $\dfrac{32a^5b^5}{243c^5x^5}$. 18. $\dfrac{2(a-b)}{3b}$.

40. $\frac{2bc}{a^2-b^2}$.

41. $\frac{1}{x-y}$.

42. $\frac{12a}{1-9a^2}$.

43. $\frac{a+x}{ax}$.

44. $\frac{a+b}{2(a-b)}$.

45. $\frac{6x}{2a-3x}$.

46. $\frac{3a^2-2ab+3b^2}{2(a^2-b^2)}$.

47. $\frac{3x+2}{x-1}$.

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49. $\frac{2xa^n}{a^{2n}-1}$.

50. $\frac{4a}{a+x}$.

51. $\frac{b(a+b)}{x^2-b^2}$.

52. $\frac{b}{a-b}$.

53. $\frac{2x-3}{x(4x^2-1)}$.

54. $\frac{2a+b}{2a-b}$.

55. $\frac{2}{x(1-4x^2)}$.

56. $\frac{16}{(x-2)(x+2)^2}$.

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64. $\frac{2}{x^4-y^4}$.

65. $\frac{18(2a+15)}{16a^4-81}$.

66. $\frac{x+c}{(x-a)(b-x)}$.

67. 0.

68. 0.

69. $\frac{x^2-4ax-a^2}{(x^2-a^2)^2}$.

70. 0.

71. 0.

72. -1.

73. $\frac{1}{abc}$.

74. 0.

75. $\frac{2x^2-9x+44}{x^3+64}$.

76. $\frac{2a}{x^2-a^2}$.

77. $\frac{x^3+1}{x^4-3x^3+3x^2-x}$.

78. 1.

79. $\frac{x(x-2)}{x^3+1}$.

80. $\frac{2x^2}{x^3+y^3}$.

81. $\frac{2y^2}{x^3-y^3}$.

82. $\frac{4-3x+2x^3-3x^4}{2(1+x^2)(1+x^3)}$.

83. $\frac{2(x+1)}{x^2+x+1}$.

84. $\frac{3x^2+9}{x^3-27}$.

85. $\dfrac{4a^3}{a^8+a^4+1}$.

86. 0.

87. $-\dfrac{2(3x+4)}{(x-1)^2}$.

88. $-\dfrac{2x^4+10x^3-9x^2-16x+4}{(4-x^2)^2}$.

89. $\dfrac{9x^2+1}{(2x-1)^3}$.

90. $\dfrac{7a^2-4ax-2x^2}{2(a-2x)^3}$.

91. $\dfrac{4}{x+2}$.

92. $\dfrac{5a}{x-5a}$.

93. $\dfrac{a(x-9a)}{(x-6a)(x-4a)}$.

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95. $\dfrac{84-186x+93x^2-6x^3}{4(3-2x)^2(3+2x)}$.

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97. $\dfrac{4x^3+40x^2+70x-22}{x^4+5x^3-13x^2-77x-60}$.

98. $\dfrac{11-x^2}{(x-2)(x-5)(x-7)}$.

99. $\dfrac{3a^2-41ax+74x^2}{3(2a-5x)^3}$.

100. 0.

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1. $\dfrac{18p^2q^9r^8}{35m^2x^{11}y^{13}}$.

2. $\dfrac{8x^3}{3yz^2}$.

3. $\dfrac{4d}{9acq}$.

4. 1.

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6. $\dfrac{3c^2}{8}$.

7. $\dfrac{3a^2}{5bc}$.

8. $\dfrac{2m^{12}n}{a^2b^{25}}$.

9. $\dfrac{a^2}{b^2}$.

10. $\dfrac{8a^3}{125b^3}$.

11. $\dfrac{625x^4y^4}{2401a^4b^4}$.

12. $\dfrac{32a^5b^5}{243c^5x^5}$.

13. $\dfrac{8a^6b^3x^{21}}{27y^{12}}$.

14. $\dfrac{a^{18}y^{2n}x^{3n-9}}{32b^{3n-4}z^{8}}$.

15. $\dfrac{1}{9a^4x^2}$.

16. $\dfrac{9a^{10}c^8x}{2b^{11}}$.

17. $\dfrac{16b^3}{27d^3}$.

18. $\dfrac{2(a-b)}{3b}$.

40. $\frac{2bc}{a^2-b^2}$.

41. $\frac{1}{x-y}$.

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66. $\frac{x+c}{(x-a)(b-x)}$.

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68. 0.

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85. $\frac{4a^3}{a^8+a^4+1}$. 86. 0. 91. $\frac{4}{x+2}$.

87. $-\frac{2(3x+4)}{(x-1)^2}$. 92. $\frac{5a}{x-5a}$.

88. $-\frac{2x^4+10x^3-9x^2-16x+4}{(4-x^2)^2}$. 93. $\frac{a(x-9a)}{(x-6a)(x-4a)}$.

89. $\frac{9x^2+1}{(2x-1)^3}$. 94. $\frac{12+25x-x^2}{(3+x^2)(3+x)^2}$.

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100. 0.

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5. $\frac{ab^2}{y^2}$. 11. $\frac{625x^4y^4}{2401a^4b^4}$. 17. $\frac{16b^3}{27d^3}$

6. $\frac{3c^2}{8}$. 12. $\frac{32a^5b^5}{243c^5x^5}$. 18. $\frac{2(a-b)}{3b}$.

212. $(x+2a)(x+3b)$.
213. $(a+2b+c)(2b+c-a)(c+a-2b)(a+2b-c)$.
214. $3x^2y^2z^2(x^2y-2y^2z+4xz^2)$.
215. $(x-2a)(x-b)$.
216. $7(2x-1)(2x-5)$.
217. $(x^2-3xy-y^2)(x^2+3xy-y^2)$.
218. $(x^2-13x-1)(x^2+13x-1)$.
219. $(a+b)^2(a-b)^2$.
220. $3ab(b-a)$.
221. $(x-12)(x+25)$.
222. $5(1-7x^2y^2)(1-7x^2y^2)$.
223. $\frac{5}{9}(3x-10y)(3x+y)$.
224. $(x^2+13x+1)(x^2-13x+1)$.
225. $(x+1)(x-1)(x+3)(x-3)$.
226. $3y^2(x-y)(3x+2y)$.
227. $(x+a+b)(x+a-b)$.
228. $5(x-1)(x+1)$.
229. $(x-y)(x+10y)$.
230. $(7xy-1)(4xy+1)$.
231. $(x+2)(x^2+4)$.
232. $4abxy$.
233. $(7x+y)(7x+6y)$.
234. $(1+ab)(1+18ab)$.
235. $(x+1)(ax-b)$.
236. $(x-2y)(x-6y)$.
237. $(x^2+y^2)(x+y)(x-y)(x^2+9y^2)(x+3y)(x-3y)$.
238. $(x^3-20)(x^3+31)$.
239. $(a^2+b^2)(a-2b)$.
240. $14(2x-3)(x+2)$.
241. $7(x^5-1)(x^5-2)$.
242. $(3x+5y)(7x-3y)$.
243. $(x-ca-ab)(x+bc+ca)$.
244. $(ab-c^2)(ac-b^2)$.
245. $(x^2-6y^2)(13x^2-40y^2)$.
246. $(ax+y)(x+ay)$.
247. $(x^2+mxy+y^2)(x^2-mxy+y^2)$.
248. $x(x-a)(x-b)$.
249. $(x+7)(x-2)(x+2)(x+3)$.
250. $[x^2+(1+m)y^2][x^2+(1-m)y^2]$.
251. $(9x^2-11y)(11x^2+12y)$.
252. $(x^2y-z^2)(x^2y-16z^2)$.
253. $(x-ab-4c)(x+ab-3c)$.
254. $(x^2+mxy-y^2)(x^2-mxy-y^2)$.
255. $(ax-by)(bx-ay)$.

256. $a(a-2b+3x)(a-2b-3x)$.

257. $(ax^2+bx+c)(ax^2-bx+c)$.

258. $(x-a)(x+a)(x-b)(x+b)$.

259. $(x+yz)(y+zx)$.

260. $(x+y-10z)(x+y+3z)$.

261. $(ax-c)(bx+c)$.

262. $(x+1)(x+2)(x+4)(x+5)$.

263. $3x(3x+5)$.

264. $(x^2-a^2-ab-b^2)(x^2-ab+a^2+b^2)$.

265. $(5x^2-3y^6)(9x^2+25y^6)$.

266. $(2a-b)(a^2+2bx)$.

267. $(x-a+b)(x+a+b)$.

268. $(x-b+a)(x-b-a)$.

269. $11(3x-4)(3x-5)$.

270. $ab(a^2-2ab-2b^2)$.

271. $(x^4y^2+3)(x^4y^2+12)$.

272. $12(4x^2-11)(4x^2+1)$.

273. $(x^2+y^2)(x+8y)(x-8y)$.

274. $(x-3y^2)(11x-21y^2)$.

275. $(x^2+4y)(x^2+9y)$.

276. $3(x+2y)(x+15y)$.

277. $(a-b)(a^2-2b^2)$.

278. $\frac{4}{15}(4a-b)(4b-a)$.

279. $x^2(6a+x)(4a-3x)$.

280. $(3x+2)(x^3-7)$.

281. $a^3(a^2+ab-b^2)(a^2-ab+b^2)$.

282. $(ax+y)(bx-y)$.

283. $(x-b)(x+b)(x-a^2)$.

284. $(a^4+b^4)(a^2+b^2)^2(a+b)^2(a-b)^2$.

285. $(x-a)(x+a)(x+b)$.

286. $(x+y)(x-y)^3$.

287. $(2+ax)(a-2x)$.

288. $5(x^4-25)+4x(x^2-5)$
$=(x^2-5)(5x^2+4x+25)$.

289. $(a+bx)(a-bx+cx^2)$.

290. $x^2(6x-7)(6x+5)$.

291. $2a(a^2+3b^2)$.

Ex. 9. Page 31.

1. $2(x+1)$.
2. $9(a+b)$.
3. $3(x+1)$.
4. $4(x^2+y^2)$.
5. $a-x$.
6. $ax(a-x)^2$.
7. a^2-b^2.
8. $x-y$.
9. $3(a^2c+2b^2)$.
10. $2a+3b$.
11. $b-a$.
12. $2a+5$.
13. $4a+3b$.
14. $5x-1$.
15. $x^5(3a^3-4ax^2y^2-6x^3y)$.
16. $x+y$.
17. $(a+b)^2$.
18. $2x-3y$.
19. $a+b$.
20. $x+5$.
21. $x-7$.
22. $x-10$.
23. $x-12$.
24. $x+1$.
25. $3x-2$.
26. $3x-2a$.
27. $2x+3$.
28. $x-1$.
29. $x(x+3)$.
30. $2(7-3x)$.
31. $x+1$.
32. x^2+7.
33. $x-y$.
34. $a+5$.
35. $x-y$.
36. $a-b$.
37. $x-a$.
38. $(x-y)^2$.
39. $5x^2-1$.
40. x^2-4x+4.
41. x^2-4y^2.
42. $a(a^2-2a+1)$.
43. $9x^2(x-1)$.
44. x^2-y^2.
45. a^3+a^2-5a+3.
46. $x+3$.
47. a^2+b^2.
48. $a(2a^2+2ab-b^2)$.
49. $7x^2+8x+1$.
50. $x^4-2x^3+3x^2$
$-2x+1$.
51. x^2-3x+1.
52. $x+1$.
53. $a(2a-3x)$.
54. $(x+1)^3$.
55. $2x^3-4x^2+x-1$.
56. $x-2a$.
57. $x+a$.
58. $x^2-(a+b)x+ab$.
59. $2x-9$.
60. x^2-1.
61. $x^2-(a-b)x+b^2$.
62. x^3+x^2+x+1.
63. $x+1$.
64. $x+a$.
65. $a-b$.
66. $x-2a$.
67. $a+b$.
68. $3x-y$.

Ex. 10. Page 34.

1. $100a^5b^4c^6$.
2. $ab(a+b)$.
3. $12ab(b-1)$.
4. $90b(a^2-b^2)$.
5. $(a+b)(c^2-d^2)$.
6. $(a+b)(a-b)^2$.
7. $12(a^2-x^2)$.
8. $ab(a^2x^2-b^2y^2)$.
9. $b^2(b+ax)$.

10. $(x+1)(x+3)(x-4)$.
11. x^3+3x^2-x-3.
12. $12x^3-8x^2-27x+18$.
13. $x(2x+1)(3x-1)(4x+3)$.
14. $(x+2)(x+4)(x^2+3x+1)$.
15. $36x^3-3ax^2-50a^2x+24a^3$, or $(4x-3a)(3x-2a)(3x+4a)$.
16. $(a^3-9a^2+23a-15)(a-7)$.
17. $(a^2-b^2)(a^2-4b^2)$.
18. $60x^6y^2+190x^5y^3+116x^4y^4+64x^3y^5+48x^2y^6-30xy^7$.
19. a^6-b^6.
20. $mnpq$.
21. $36x^3y^3z^3$.
22. x^2-y^2.
23. $(x+y)(x-y)^2$.
24. $6a(a^2+3a+2)$.
25. $a(a^2-4b^2)$.
26. $a(4a^2-1)$.
27. $2a(a^2-b^2)$.
28. x^3-1.
29. $(x+1)(x+2)(x+3)$.
30. $(x^3-19x-30)(x^2+5x+10)$.
31. $(x+1)(x+2)(x+3)(x-1)$.
32. $(4x^2-9)(9x^2-4)$.
33. $(a-1)(a-2)(a-3)(a-4)$.
34. x^8-1.
35. $x^{12}-1$.
36. $(x-a)(x-b)(x-c)$.
37. $(x+c)(2x-3b)(x^2+ax-b^2)$.

Ex. 11. Page 35.

1. $\dfrac{3n^3}{2q}$.
2. $\dfrac{2a}{3b}$.
3. $\dfrac{3b^2}{5ac}$.
4. $\dfrac{6a^2}{5x^2}$.
5. $\dfrac{4a}{7b^2c^2}$.
6. $\dfrac{4ab}{9x}$.
7. $\dfrac{7b^2-4ac}{6c^2-7ab}$.
8. $\dfrac{8a^2b-x^3}{9a^3-b^2x}$.
9. $\dfrac{2x^4-3a^2b^2}{3bx^3-4a^4}$.
10. $\dfrac{a+b}{a(1-3c^2)}$.
11. $\dfrac{b^2-3ac}{c^2-4ab}$.
12. $\dfrac{2x^2-y}{3x^2-y^2}$.
13. $\dfrac{2z^2-3y^2}{3z^2-2y^2}$.
14. $\dfrac{3c^2-a^2}{2a^2+c^2}$.
15. $\dfrac{3a^2-7c}{4a^2+9b^2}$.

16. $\frac{5a^2-3b}{2(2b^2-3a)}$.
17. $\frac{1}{2x+1}$.
18. $\frac{1}{3x-1}$.
19. $\frac{1}{8x^3-1}$.
20. $\frac{x^2}{x-3}$.
21. $\frac{xy}{x+1}$.
22. $\frac{x^2-y}{x}$.
23. $\frac{x(3x^2+2)}{2y^2}$.
24. $\frac{x^2y^2}{x^2z^2+y^4}$.
25. $\frac{3x}{4(a-3x)}$.
26. $\frac{x^2}{x+y}$.
27. $\frac{x^2}{x+2}$.
28. $\frac{4x}{3x+2y}$.
29. $\frac{3b^2}{a^2-2c}$.
30. $\frac{1}{5x(7x^2+3)}$.
31. $\frac{3a}{5a-9b}$.
32. $\frac{(a-b)(a^2+b^2)}{a+b}$.
33. $\frac{x+z}{(1-y)^2}$.
34. $\frac{ax}{(a+b)^2}$.
35. $\frac{4a^3x^2}{3b(5a^2+4b)}$.
36. $\frac{x^2-ax+b^2}{x^2+ax-b^2}$.
37. $\frac{x+c}{a+b-x}$.
38. $\frac{x-a}{x-b+c}$.
39. $\frac{x-b}{bc-a(c+x)}$.
40. $\frac{bc}{(a+b+c)(b+c-a)}$.
41. $\frac{x+y+z}{x-y-z}$.
42. $\frac{x+2}{x+5}$.
43. $\frac{x+7}{x-5}$.
44. $\frac{x-2}{x-3}$.
45. $\frac{x-2}{x+1}$.
46. $\frac{x-4}{x-5}$.
47. $\frac{x+4}{x+6}$.
48. $\frac{a-7b}{a+b}$.
49. $\frac{a+b}{a-b}$.
50. $\frac{x+3}{x-3}$.
51. $\frac{2x-3}{7x-3}$.
52. $\frac{4a-3b}{5a+2b}$.
53. $\frac{2x-1}{x-1}$.
54. $\frac{3x-4y}{5x-3y}$.
55. $\frac{4y+3}{3(y+2)}$.
56. $\frac{(2a+1)x}{a+1}$.
57. $\frac{(x-1)^2}{x^2-3x+1}$.
58. $\frac{a^2+b^2}{a}$.
59. $\frac{1}{b-2x}$.
60. $\frac{7(x^2+xy+y^2)}{5}$.

Ex. 12. Page 38.

1. $\frac{a+2m^2}{m}$.
2. $\frac{3x-a}{x-a}$.
3. $\frac{4b}{3a+2b}$.
4. $\frac{3a^2-2b^2}{6ab}$.
5. $\frac{5ay-2bx}{x^2y}$.
6. $\frac{x^2-2y^2}{10xy}$.
7. $\frac{5n+ax}{5a^2}$.
8. 2.
9. $\frac{b^2-a^2}{ab}$.
10. $\frac{3a+b}{2b}$.
11. $\frac{4a+3b}{12a}$.
12. $\frac{21a-16b}{6}$.
13. $\frac{133a}{36}$.
14. $\frac{2a+3b}{b}$.
15. $\frac{x+y}{3}$.
16. $\frac{a-b}{2}$.
17. $-\frac{x+5y}{15}$.
18. $\frac{a}{12}$.
19. 0.
20. $\frac{4a}{45}$.
21. $\frac{3b+5c-5a}{36}$.
22. $\frac{11a-8b+3c}{84}$.
23. 0.
24. $a+b$.
25. $\frac{5a+286b}{30b}$.
26. $\frac{ab+ac+bc}{abc}$.
27. $\frac{a(np+mp+mn)}{mnp}$.
28. $\frac{az-by-cx}{xyz}$.
29. $\frac{2ab+3bx-4ax}{abx^2}$.
30. $\frac{7}{60b}$.
31. $\frac{a}{6x}$
32. $\frac{a(3df-5cf+4cd)}{bcdf}$.
33. $\frac{2(9adg+60ad-5cg)}{45bdg}$.
34. $\frac{a}{18x}$.
35. $\frac{ab+ac+bc}{abc}$.
36. $\frac{a^2+b^2+c^2}{abc}$.
37. $\frac{a^2-x^2}{2a^2x^2}$.
38. $\frac{2a}{a^2-b^2}$.
39. $\frac{a^2+2ab-b^2}{a^2-b^2}$.

40. $\frac{2bc}{a^2-b^2}$.

41. $\frac{1}{x-y}$.

42. $\frac{12a}{1-9a^2}$.

43. $\frac{a+x}{ax}$.

44. $\frac{a+b}{2(a-b)}$.

45. $\frac{6x}{2a-3x}$.

46. $\frac{3a^2-2ab+3b^2}{2(a^2-b^2)}$.

47. $\frac{3x+2}{x-1}$.

48. $\frac{6ax}{a-x}$.

49. $\frac{2xa^n}{a^{2n}-1}$.

50. $\frac{4a}{a+x}$.

51. $\frac{b(a+b)}{x^2-b^2}$.

52. $\frac{b}{a-b}$.

53. $\frac{2x-3}{x(4x^2-1)}$.

54. $\frac{2a+b}{2a-b}$.

55. $\frac{2}{x(1-4x^2)}$.

56. $\frac{16}{(x-2)(x+2)^2}$.

57. $\frac{a^4+6a^2x^2+x^4}{a^4-x^4}$.

58. $\frac{2}{(x+1)(x+2)(x+3)}$.

59. $\frac{x(5x-7)}{(x^2-1)(x-2)}$.

60. $\frac{2x-3}{(x^2-1)(2x+3)}$.

61. $\frac{8x^4-x^3-24x^2+5x-4}{(x^2-1)(x+3)(x-2)}$.

62. $\frac{3a^4+4a^3x+6a^2x^2-4ax^3+3x^4}{a^4-x^4}$.

63. $\frac{2x^2+21x+13}{(1-x^2)(1-4x^2)}$.

64. $\frac{2}{x^4-y^4}$.

65. $\frac{18(2a+15)}{16a^4-81}$.

66. $\frac{x+c}{(x-a)(b-x)}$.

67. 0.

68. 0.

69. $\frac{x^2-4ax-a^2}{(x^2-a^2)^2}$.

70. 0.

71. 0.

72. -1.

73. $\frac{1}{abc}$.

74. 0.

75. $\frac{2x^2-9x+44}{x^3+64}$.

76. $\frac{2a}{x^2-a^2}$.

77. $\frac{x^3+1}{x^4-3x^3+3x^2-x}$.

78. 1.

79. $\frac{x(x-2)}{x^3+1}$.

80. $\frac{2x^3}{x^3+y^3}$.

81. $\frac{2y^3}{x^3-y^3}$.

82. $\frac{4-3x+2x^3-3x^4}{2(1+x^2)(1+x^3)}$.

83. $\frac{2(x+1)}{x^2+x+1}$.

84. $\frac{3x^2+9}{x^3-27}$.

85. $\frac{4a^3}{a^8+a^4+1}$.
86. 0.
87. $-\frac{2(3x+4)}{(x-1)^2}$.
88. $-\frac{2x^4+10x^3-9x^2-16x+4}{(4-x^2)^2}$.
89. $\frac{9x^2+1}{(2x-1)^3}$.
90. $\frac{7a^2-4ax-2x^2}{2(a-2x)^3}$.
91. $\frac{4}{x+2}$.
92. $\frac{5a}{x-5a}$.
93. $\frac{a(x-9a)}{(x-6a)(x-4a)}$.
94. $\frac{12+25x-x^2}{(3+x^2)(3+x)^2}$.
95. $\frac{84-186x+93x^2-6x^3}{4(3-2x)^2(3+2x)}$.
96. $\frac{(z^4-2z^2-3)(4z^2-3)-(z^2-4z+1)(5z^4+5z^2-9)}{(4z^2-3)(3z^2+2)(5z^4+5z^2-9)}$.
97. $\frac{4x^3+40x^2+70x-22}{x^4+5x^3-13x^2-77x-60}$.
98. $\frac{11-x^2}{(x-2)(x-5)(x-7)}$.
99. $\frac{3a^2-41ax+74x^2}{3(2a-5x)^3}$.
100. 0.

Ex. 13. Page 43.

1. $\frac{18p^2q^9r^8}{35m^2x^{11}y^{13}}$.
2. $\frac{8x^3}{3yz^2}$.
3. $\frac{4d}{9acq}$.
4. 1.
5. $\frac{ab^2}{y^2}$.
6. $\frac{3c^2}{8}$.
7. $\frac{3a^9}{5bc}$.
8. $\frac{2m^{12}n}{a^2b^{25}}$.
9. $\frac{a^2}{b^2}$.
10. $\frac{8a^3}{125b^3}$.
11. $\frac{625x^4y^4}{2401a^4b^4}$.
12. $\frac{32a^5b^5}{243c^5x^5}$.
13. $\frac{8a^6b^9x^{12}}{27y^{15}}$.
14. $\frac{a^{10}y^{5p}x^{5n-15}}{32b^{5x-5}z^{20}}$.
15. $\frac{1}{9a^5x^2}$.
16. $\frac{9a^{10}c^6x}{2b^{11}}$.
17. $\frac{16b^3}{27d^3}$.
18. $\frac{2(a-b)}{3b}$.

19. $\frac{4x(x-y)}{x^2+y}$.

20. $\frac{25b^6c^{10}}{1024a^{20}}$.

21. $\frac{2}{5}$. **22.** $\frac{4y}{x-1}$.

23. $-\frac{3x^2(x-y)}{4(x^2+y^2)}$.

24. $\frac{a^2}{d^2}$.

25. $\frac{20}{21}$.

26. $\frac{12ab}{5f(a-b)}$.

27. $\frac{3x-1}{x-2}$.

28. $\frac{9}{20x}$.

29. $\frac{y(x-y)}{x(x+y)}$.

30. $\frac{x^2+y^2}{x}$.

31. $\frac{ax}{a^2-x^2}$.

32. $\frac{(a+b)^2}{a^2+b^2}$.

33. $\frac{x+p}{x+n}$.

34. $\frac{a^2-ab+b^2}{a^2+ab+b^2}$.

35. $\frac{a+b}{a-b}$.

36. $\frac{a(2a+5x)(2a^2+19ax+42x^2)}{x(a-x)(a^2+2ax+2x^2)}$.

37. $\frac{(a+b)^2}{ab}$, or $2+\frac{b}{a}+\frac{a}{b}$.

38. $2b+\frac{3b^2}{a}-\frac{2a^2}{b}-3a$.

39. $\frac{1}{a^2}-\frac{1}{b^2}$.

40. $\frac{ad}{bc}-\frac{145}{72}+\frac{bc}{ad}$.

41. $\frac{x}{x-y}$.

42. $\frac{4ab}{a^2-b^2}$.

43. $\frac{a^2(a-b)}{x}$.

44. $\frac{(a-c)^2-b^2}{abc}$.

45. $\frac{x^6-ax^5+a^5x-a^6}{a^3x^3}$.

46. $\frac{x^2}{a^2}+\frac{a^2}{x^2}-\frac{y^2}{b^2}-\frac{b^2}{y^2}$.

47. $\frac{4ab}{c^2x^3y^5}$.

48. $\frac{15a^nx^n}{28b^my^m}$.

49. $\frac{7b^2x^{n-3}}{3a^nc^2}$.

50. $\frac{a^3z^7}{b^5y^4}$.

51. $\frac{xy^2z^3}{a^pb^mc^n}$.

52. $\frac{21pqxy}{5mnzu}$.

53. $\frac{77abxy}{15mnpq}$.

54. $\frac{6a^6b^8c^3}{65d^9f^9g^{14}}$.

55. $\dfrac{55q^4y^2}{42p^3r^2x^3z^4}$.

56. -1.

57. $-\dfrac{2}{3}$.

58. $\dfrac{1}{3(x-y)}$.

59. $\dfrac{3(a-b)^2}{b}$.

60. $\dfrac{20b(y+z)}{3(x-y)}$.

61. $\dfrac{x(a+2x)}{a^2}$.

62. $\dfrac{2}{a-b}$.

63. $\dfrac{x+y}{a+b}$.

64. $\dfrac{y-b}{y-a}$.

65. $\dfrac{x+y-z}{x-y+z}$.

66. $\dfrac{1}{a^2-b^2}$.

67. $\dfrac{(a-2)^2}{(a-3)(a-1)}$.

68. $\dfrac{x+y}{a-b}$.

69. $-\dfrac{y^2}{m^2}$.

70. $\dfrac{a-b}{b}$.

71. $\dfrac{b(a-x)}{a(a-b)}$.

72. $-\dfrac{a(x+2)}{c(x+1)}$.

74. $\dfrac{p^2+3}{p-3}$.

73. $\dfrac{8m^2(4m-3n)(2m-9n)}{27n^2(2m+9n)(4m+3n)}$.

Ex. 14. Page 47.

1. $\dfrac{x+y}{z}$.

2. $\dfrac{m-c}{a+d}$.

3. $\dfrac{3mn-mp}{5np}$.

4. $-\dfrac{5n}{12}$.

5. $\dfrac{3x-2y}{3z-u}$.

6. $\dfrac{x}{z}$.

7. $-\dfrac{d}{c}$.

8. $\dfrac{a+1}{a-1}$.

9. $\dfrac{x+y}{x-y}$.

10. $\dfrac{m+n}{m-n}$.

11. $\dfrac{2xy}{x^2+y^2}$.

12. $\dfrac{mq+np}{oq-nt}$.

13. $\dfrac{y}{x}$.

14. y.

15. $\dfrac{a-4}{a-5}$.

16. 1.

17. 1.

18. $\dfrac{1}{x+1}$.

19. $\dfrac{y^2-a^2}{y(a+b+c)-bc}$.

20. $\dfrac{1}{a+1}$.

21. $\dfrac{1+a}{1+a^2}$.

22. $a+1$.

23. $\dfrac{1+a^2}{1+a}$.

24. $\frac{4}{3(a+1)}$.

25. $\frac{adf+ae}{bdf+be+cf}$.

26. 1.

27. $\frac{bc+ac+ab}{bc+ac-ab}$.

28. $-\frac{m^4+m^2n^2+n^4}{mn(m-n)^2}$.

29. $-\frac{xy(y-x)^2}{x^4+y^4+x^2y^2}$.

30. $\frac{ab}{a^2+b^2}$.

31. $\frac{(m^2+n^2)^2}{2m^2n^2}$.

32. a.

33. $\frac{4xy^3}{x^4-y^4}$.

34. $\frac{(a+b+c)^2}{2bc}$.

35. $\frac{12m}{5n}$.

36. 3.

37. 1.

38. $\frac{a^2-b^2}{16a^2b^2}$.

39. $\frac{x-y}{xy}$.

40. $\frac{4}{3}$.

41. $\frac{2(1+x^4)}{x}$.

42. $\frac{3+x^2}{x(1-x^2)}$.

43. 1.

Ex. 15. Page 51.

1. $\frac{0}{a}=0$.

2. $\frac{1}{\infty}=0$.

3. $\frac{n}{0}=\infty$.

4. $\frac{0}{0}$.

5. $\frac{0}{0}$.

6. $\frac{3}{7}$.

7. $\frac{1}{2}$.

8. $\frac{3}{4}$.

9. -1.

10. 32.

11. $\frac{2}{3}$.

12. 0.

13. ∞.

14. $\frac{3}{5}$.

15. 0.

16. $\frac{3a}{2}$.

17. $\frac{1}{4}$ and $\frac{2}{5}$.

18. 4.

19. 4.

20. $\frac{4}{5}$.

21. $\frac{9}{8}$.

22. 0.

23. $\frac{7}{12}$.

24. $\frac{7}{5}$.

25. $\frac{1}{8}$.

26. $\frac{11}{17}$.

27. $\frac{1}{2}$.

28. $-\frac{1}{2}$.

29. 5.

30. $\frac{24-a^3}{3a^2+16}$.

31. $\frac{a}{b}$.

32. $\frac{3}{2}$.

Ex. 16. Page 53.

1. $\frac{m^2}{n^2}$.

2. $\frac{n}{m}$.

3. 0.

4. 0.

5. a.

6. 0.

7. $\frac{2x}{1+x^2}$.

8. b^3.

9. 2.

10. $\frac{a}{m-\frac{1}{b}}=\frac{a}{m}$.

11. $\frac{1-x^2}{1+x^2}$.

12. 0.

13. 1.

19. $\frac{(a+b-c)(a-b+c)}{2bc}$.

20. x^{n-1}; remaining fraction, $\frac{x^n}{1-x}$.

21. $\frac{1}{0.97}=\frac{1}{1-0.03}=1+0.03+0.0009+0.000027$
$+0.00000081=1.0309278.$

22. $\frac{3}{107}=\frac{3}{100+7}=0.03-0.0021+0.000147$
$-0.00001029+0.00000072=0.0280374.$

23. $\frac{3}{104}=\frac{3}{100+4}=0.03-0.0012+0.000048$
$-0.00000192=0.0288461.$

24. $\frac{5}{0.98}=5\left(\frac{1}{1-0.02}\right)=5(1+0.02+0.0004+0.000008$
$+0.00000016+0.0000000032)=5.10204082.$

25. $\frac{7}{0.995}=7\left(\frac{1}{1-0.005}\right)=7(1+0.005+0.000025+\cdots\cdots)$
$=7.0351758793968.$

26. $\frac{3}{1001}=0.00299700299700.$

27. 0.8787·····

Ex. 17. Page 56.

1. -12.
2. $-\frac{2}{11}$.
3. $-52\frac{3}{20}$.
4. 0.
5. 1.
6. $\frac{1}{4}$.
7. 4.
8. 5.
9. 3.
10. 2.
11. 7.
12. 3.
13. 1.
14. 5.
15. 7.
16. $2\frac{8}{9}$.
17. 4.
18. 0.
19. 8.
20. $-1\frac{76}{115}$.
21. 1.
22. 2.
23. 1.
24. 10.
25. $-1\frac{20}{91}$.
26. 90.
27. 28.
28. 12.
29. 70.
30. 12.
31. 28.
32. 36.
33. 20.
34. 4.
35. 6.
36. 11.
37. 1.
38. -5.
39. $7\frac{4}{61}$.
40. 8.
41. 2.
42. 3.
43. 12.
44. $\frac{6}{7}$.
45. 10.
46. 5.
47. 6.
48. 1.
49. 19.
50. $1\frac{57}{385}$.
51. $3\frac{5}{21}$.
52. 10.
53. $-10\frac{17}{37}$.
54. 13.
55. $23\frac{5}{6}$.
56. 3.
57. 11.
58. 5.
59. 28.
60. 7.
61. $\frac{4}{13}$.
62. 4.
63. 17.
64. $84\frac{2669}{2671}$.
65. $-\frac{427}{1739}$.
66. 1.
67. $-22\frac{1}{4}$.
68. $10\frac{195}{544}$.
69. -6.
70. $\frac{301}{401}$.
71. $\frac{5}{18}$.
72. 8.
73. 7.
74. 36.
75. 6.
76. $-\frac{2803}{4810}$.
77. $1\frac{117}{163}$.
78. 2.43466.....
79. 6.
80. 36.
81. 3.
82. 24.
83. 4.
84. 17.
85. 13.
86. 12.
87. 3.
88. $\frac{45}{223}$.
89. $2\frac{1}{2}$.
90. $2\frac{1}{2}$.
91. $\frac{95}{37}$.
92. $\frac{6}{13}$.
93. 2.
94. -7.
95. 6.
96. 7.
97. 17.
98. 6.
99. 2.
100. 0.
101. 2.
102. 4.
103. $\frac{1}{2}$.
104. 8.
105. 2.
106. 1.
107. $\frac{83}{84}$.
108. 7.
109. 7.
110. 3.

Ex. 18. Page 62.

1. $\frac{k-m}{p(n-q)}$.
2. $\frac{db-ag}{bg(c-3)}$.
3. $\frac{2d}{a+b}$.
4. $\frac{q(m+n)-p(c-d)}{pq}$.
5. $\frac{m(m-n+1)}{m+n+1}$.
6. $\frac{p}{m}$.
7. $\frac{ab}{c}$.
8. $\frac{bm-cn-dn}{b}$.
9. $\frac{1-m}{1+m}$.
10. $\frac{bp-m}{n-bq}$.
11. $\frac{n(a+b)}{m(a-b)}$.
12. $\frac{4m-3n}{5}$.
13. 0.
14. 0.
15. $\frac{2m}{n}$.
16. $\frac{np}{m}$.
17. 1.
18. 1.
19. 0.
20. $\frac{a^2-b^2}{b-4a}$.
21. $\frac{abcd}{ab+ac+bc}$.
22. $\frac{bdgh}{adg-bcg+bdf}$.
23. $\frac{a^2bc^2+ab+c^2}{a^2-bc}$.
24. $\frac{c(-a^3b+a^2+ab^2+ab-b^2)}{ab+ac-bc}$.
25. $\frac{b^2+c^2-ac+b^2c}{b+c}$.
26. $\frac{15a-4b+3abc}{a-5b+abd}$.
27. 0.
28. 0.
29. n.
30. 1.
31. $\frac{a^2b}{a-c}$.
32. $a-b$.
33. $\frac{0.28bc^2d^2}{d^2-0.2bc}$.
34. $\frac{2b^3+12ab^2-9a^3}{6a^2+10b^2}$.
35. $\frac{5(2a^2b+2b^3)-15a^3}{4b(10a-3b)}$.
36. $\frac{n}{m}$.
37. $\frac{mq}{anp}$.
38. $\frac{cfn+bdp}{adp-cfm}$.
39. $\frac{b}{b-a-1}$.
40. $\frac{mq(m+n)}{p}$.
41. $\frac{n(n^2+m^2)}{m(n^2-m^2)}$.
42. $\frac{d}{c}$.

43. $\frac{a(b-c)+3(b+c)}{b+c}$.

44. $mn+mp+np$.

45. $\frac{mnp}{m+n+p}$.

46. $\frac{3a(a-b)}{8b-3a}$.

47. $\frac{2(15a^2+17ab+4b^2)}{25a}$.

48. $\frac{3(8a^2+6ab-b^2)}{17a-6b}$.

49. $5a$.

50. $4a$.

51. $\frac{3d^3-2c^3}{12cd}$.

52. $\frac{n}{n-1}$.

53. $\frac{b(a-b+c)}{a}$.

54. $\frac{a^2(b-a)}{b(b+a)}$.

55. $\frac{c^2-ab}{a+b-2c}$.

56. $\frac{mn}{m+n-p}$.

57. $\frac{mn(m+n-2p)}{(m+n)p-m^2-n^2}$.

58. $\frac{2mn}{m+n}$.

59. $\frac{a+b+c+d}{m+n}$.

60. $\frac{c^2}{d-c}$.

61. $\frac{ab}{a+b}$.

62. $5a$.

63. $\frac{5a(a+b)}{2(a+4b)}$.

64. $\frac{81m(n-m)}{56n}$.

65. $\frac{a-3b}{3}$.

66. $\frac{a}{6}$.

67. $m+n$.

68. $\frac{1}{p}+\frac{1}{q}$.

69. p.

70. q.

71. $\frac{p(s-m)}{pn+pr-qs}$.

72. $\frac{de}{d+e}$.

73. $4k$.

74. $\frac{f(4f^2-9g^2)}{8f^2+27g^2}$.

75. $\frac{4a^2(ab-b^2+a^2)}{3a^3-6a^2b+ab^2+6b^3}$.

Ex. 19. Page 66.

1. 160,000 inhabitants.
2. Together, $27\frac{9}{53}$ dys.; A alone, $62\frac{14}{23}$ dys.; B alone, $110\frac{10}{13}$ dys.; C alone, $84\frac{12}{17}$ dys.
3. $2.58\frac{1}{3}$ lbs.
4. 4.55 ft.
5. 3211 yds.
6. 2 hrs. 24 min.
7. $\frac{bm}{a-b}$ hrs.
8. 1188 mi.
9. 30 mi. per hour.
10. 4 mi.
11. The next day at 1.30 o'clock in the morning, and at 188 mi. from A.
12. At the end of 6 hrs., and at $31\frac{1}{2}$ mi. from A.
13. $32\frac{8}{11}$ min. past 12.
14. 10 times; $5\frac{5}{11}$ min. past 1; $10\frac{10}{11}$ min. past 2; $16\frac{4}{11}$ min. past 3; and so on.
15. 300 leaps.
16. 6780 steps.
17. Length of the route 15 mi.; speed, 2 mi. per hour.
18. $2\frac{2}{9}$ mi. per hour.
19. $\frac{n(t'-t)}{tt'(m+1)}$.
20. Together, $\frac{2mnp}{mn+mp+np}$ dys.; A alone, $\frac{2mnp}{mp+np-mn}$ dys.; B alone, $\frac{2mnp}{mn+np-mp}$ dys.;

    C alone, $\frac{2mnp}{mn+mp-np}$ dys.
21. $\frac{abn}{b-a}$ ft.
22. $\frac{pqr}{pq+q-p}$.
23. $\frac{ka}{n(n-d-a)}$ miles; $16\frac{2}{3}$ miles.
24. A's part $= \frac{ads}{ad+bd+bh}$; B's part $= \frac{bds}{ad+bd+bh}$;

    C's part $= \frac{bhs}{ad+bd+bh}$.
25. $\frac{d(b-d)}{d+1}$; 108.

26. Of the first $\frac{l(m-b)}{a-b}$ gals.; of the second, $\frac{l(a-m)}{a-b}$ gals.

27. $\frac{4a}{\pi}$.

28. $\frac{100^n A}{(100+t)^n}$.

29. A's part $= \frac{amg}{am+bn+cp}$; B's part $= \frac{bng}{am+bn+cp}$; C's part $= \frac{cpg}{am+bn+cp}$.

30. $\frac{an}{(n+1)^2} - n$; $\frac{an}{(n+1)^2} + n$; $\frac{a}{(n+1)^2}$; $\frac{an^2}{(n+1)^2}$; 18, 22, 10, 40.

31. $\frac{2mn(2m+n)}{4m^2+4mn-n^2}$ dys. **32.** $\frac{abmp}{mp-(p-n)a}$. **33.** $\frac{ab}{b-ac}$.

Ex. 20. Page 71.

1. $x = 4$, $y = 3$.
2. $x = 12$, $y = 4$.
3. $x = 8$, $y = 4$.
4. $x = 13$, $y = 3$.
5. $x = 7$, $y = 2$.
6. $x = 5$, $y = 1$.
7. $x = 7$, $y = 17$.
8. $x = 12$, $y = 9$.
9. $x = 2$, $y = 3$.
10. $x = 3$, $y = 20$.
11. $x = -2$, $y = 19$.
12. $x = -5$, $y = 14$.
13. $x = -3$, $y = -2$.
14. $x = 7$, $y = -5$.
15. $x = \frac{1}{2}$, $y = -\frac{1}{3}$.
16. $x = -2$, $y = 1$.
17. $x = 3$, $y = 4$.
18. $x = 2$, $y = 1$.
19. $x = 2$, $y = 3$.
20. $x = 8$, $y = 2$.
21. $x = 1$, $y = -1$.
22. $x = 34$, $y = 46$.
23. $x = 5$, $y = 4\frac{1}{3}$.
24. $x = 1$, $y = \frac{1}{2}$.
25. $x = -2$, $y = -8$.
26. $x = 13$, $y = 5$.
27. $x = 10$, $y = 4$.
28. $x = 12$, $y = 5$.
29. $x = 9$, $y = 8$.
30. $x = 24$, $y = 45$.
31. $x = 6$, $y = 12$.
32. $x = 1$, $y = 2$.
33. $x = 21$, $y = 35$.
34. $x = 6$, $y = 12$.
35. $x = 10$, $y = 5$.
36. $x = 20$, $y = 20$.
37. $x = 9$, $y = 7$.
38. $x = 12$, $y = 6$.
39. $x = \frac{1}{4}$, $y = \frac{1}{5}$.
40. $x = \frac{1067}{286}$, $y = \frac{278}{59}$.

41. $x=6$,
$y=5$.

42. $x=19\frac{1}{2}$,
$y=-17$.

43. $x=3$,
$y=2$.

44. $x=11$,
$y=13$.

45. $x=3$,
$y=4$.

46. $x=5$,
$y=6$.

47. $x=\frac{1}{2}$,
$y=\frac{1}{4}$.

48. $x=3$,
$y=4$.

49. $x=\frac{1}{2}$,
$y=\frac{1}{4}$.

50. $x=\frac{61}{92}$,
$y=\frac{61}{108}$.

51. $x=\frac{1}{3}$,
$y=\frac{1}{5}$.

52. $x=\frac{1}{3}$,
$y=\frac{1}{4}$.

53. $x=\frac{1}{2}$,
$y=\frac{1}{3}$.

54. $x=0.7$,
$y=0.3$.

55. $x=2$,
$y=3$.

56. $x=21$,
$y=20$.

57. $x=7\frac{368}{709}$,
$y=1\frac{269}{709}$.

58. $x=9$,
$y=7$.

59. $x=8\frac{3}{8}$,
$y=8\frac{1}{4}$.

60. $x=7$,
$y=5$.

61. $x=\frac{3}{5}$,
$y=-\frac{2}{3}$.

62. $x=4$,
$y=7$.

63. $x=-\frac{2}{9}$,
$y=\frac{1}{5}$.

64. $x=3$,
$y=2$.

65. $x=2$,
$y=3$.

66. $x=3$,
$y=2$.

67. $x=2$,
$y=2$.

68. We first obtain $\frac{xy-5}{xy-4}=\frac{xy-7}{xy-8}$, from which we find $xy=6$. The second equation gives $xy=x-1$. Introducing the value of $xy=6$, we find $x=7$, and substituting, $y=\frac{6}{7}$.

Ex. 21. Page 76.

1. $x=\frac{a+b}{2}$,
$y=\frac{a-b}{2}$.

2. $x=\frac{5(a-b)}{2}$,
$y=\frac{a+b}{2}$.

3. $x=a+b$,
$y=a-b$.

4. $x=a^2+ab+b^2$,
$y=a^2-ab+b^2$.

5. $x=\frac{cq-br}{aq-bp}$,
$y=\frac{cp-ar}{bp-aq}$.

6. $x=\frac{em+bn}{ae+bc}$,
$y=\frac{an-cm}{ae+bc}$.

7. $x=\frac{n'r+nr'}{mn'+m'n}$,
$y=\frac{mr'-m'r}{mn'+m'n}$.

8. $x=\frac{1}{ab}$,
$y=\frac{1}{cd}$.

9. $x=\frac{2b^2-6a^2+d}{3a}$,
$y=\frac{3a^2-b^2+d}{3b}$.

10. $x=\frac{abf+g}{c+af}$,
$y=\frac{a(g-bc)}{c+af}$.

11. $x = \frac{abcd}{a+bd},$
$y = \frac{abc}{a+bd}.$

12. $x = \frac{abcg}{bg-af},$
$y = \frac{abcf}{bg-af}.$

13. $x = \frac{mr(np-qs)}{nr-ms},$
$y = \frac{ns(rq-mp)}{nr-ms}.$

14. $x = \frac{n(qtr+pvs)}{ps+mqr},$
$y = \frac{qs(t-mv)}{ps+mqr}.$

15. $x = \frac{bp(dfnt-cgqs)}{gt(adnp-bcmq)},$
$y = \frac{dq(agps-bfmt)}{gt(adnp-bcmq)}.$

16. $x = \frac{(ab+ac-bc)abc}{a^2b^2+a^2c^2-b^2c^2},$
$y = \frac{(ac-ab-bc)abc}{a^2b^2+a^2c^2-b^2c^2}.$

17. $x = \frac{m}{m-n},$
$y = \frac{n}{m+n}.$

18. $x = -\frac{(a-m)(a-n)}{a-b},$
$y = \frac{(b-m)(b-n)}{a-b}.$

19. $x = \frac{a^2-b^2}{a},$
$y = \frac{a^2-b^2}{b}.$

20. $x = m+n,$
$y = m-n.$

21. $x = (p^2+1)(q^2-1),$
$y = (p^2-1)(q^2+1).$

22. $x = \frac{1+m}{1+mn},$
$y = \frac{m(1+n)}{1+mn}.$

23. $x = \frac{m^2q}{mq-np},$
$y = \frac{mnq}{mq-np}.$

24. $x = \frac{s-mq-np}{2q},$
$y = \frac{s+mq+np}{2p}.$

25. $x = \frac{(d^2-c^2)b-(a^2-b^2)c}{bd-ac},$
$y = \frac{(a^2-b^2)d-(d^2-c^2)a}{bd-ac}.$

26. $x = \frac{2}{m+n},$
$y = \frac{2}{m-n}.$

27. $x = \frac{m-n}{p-q},$
$y = \frac{m-n}{mq-np}.$

28. $x=\dfrac{ms-nr}{ps-nt},$
$y=\dfrac{ms-nr}{mt-pr}.$

29. $x=\dfrac{mnpr}{mq+nr},$
$y=\dfrac{mnpq}{mq+nr}.$

30. $x=\dfrac{a}{b},$
$y=\dfrac{c}{d}.$

31. $x=a+b-c,$
$y=a-b+c.$

32. $x=\dfrac{m^2+mn+n^2}{m+n},$
$y=\dfrac{m^2-mn+n^2}{m-n}.$

33. $x=\dfrac{m+n}{p},$
$y=\dfrac{m-n}{p}.$

34. $x=\dfrac{p+1}{mp-1},$
$y=\dfrac{m+1}{mp-1}.$

35. $x=a+b-c,$
$y=a-b+c.$

36. $x=\dfrac{a+1}{b},$
$y=\dfrac{b+1}{a}.$

37. $x=\dfrac{a-b}{a+b},$
$y=\dfrac{a+b}{a-b}.$

38. $x=\dfrac{bm}{b-m},$
$y=\dfrac{bm}{b+m}.$

39. $x=\dfrac{p+q}{p-q},$
$y=\dfrac{p-q}{p+q}.$

40. $x=\dfrac{m}{m+n},$
$y=\dfrac{n}{m-n}.$

41. $x=\dfrac{a+b}{a},$
$y=\dfrac{a-b}{b}.$

42. $x=\dfrac{m+n}{3n},$
$y=\dfrac{n-m}{2m}.$

43. $x=\dfrac{pr+q}{r},$
$y=\dfrac{pq-r}{q}.$

44. $x=\dfrac{pq+m}{q},$
$y=\dfrac{pq-m}{p}.$

45. $x = 5(c + d),$
$y = 3(c - d).$

46. $x = a(a+1)+b(b-1),$
$y = a(a-1)-b(b-1).$

47. $x = a - 2b + 3c,$
$y = 3a - 2b + c.$

48. $x = m + 3n + 2p,$
$y = 3m - n + 2p.$

49. $x = \frac{m^2 - n^2}{m},$
$y = \frac{m^2 - n^2}{n}.$

50. $x = \frac{c - d}{c + d},$
$y = \frac{c + d}{c - d}.$

51. $x = \frac{m + n^2}{2n},$
$y = \frac{m - n^2}{2n}.$

52. $x = \frac{m^2 + n^2}{2n},$
$y = \frac{m^2 - n^2}{2n}.$

53. $x = c^3 - d^3,$
$y = c^3 + d^3.$

54. $x = a^5 + b^5,\ y = a^5 - b^5.$

Ex. 22. Page 80.

1. $x = 2,$ $y = 3,$ $z = 4.$

2. $x = 1,$ $y = 3,$ $z = 5.$

3. $x = 3,$ $y = 5,$ $z = 7.$

4. $x = 1,$ $y = 2,$ $z = 3.$

5. $x = 2,$ $y = 4,$ $z = 8.$

6. $x = 10,$ $y = 20,$ $z = 30.$

7. $x = 1.7,$ $y = 1.5,$ $z = 1.3.$

8. $x = 11,$ $y = 7,$ $z = 9.$

9. $x = 12,$ $y = 18,$ $z = 35.$

10. $x = 15,$ $y = 6,$ $z = 8.$

11. $x = 3,$ $y = 2,$ $z = 1.$

12. $x = 6,$ $y = 8,$ $z = 10.$

13. $x = 11,$ $y = 12,$ $z = 13.$

14. $x = 12,$ $y = 34,$ $z = 56.$

15. $x = 50,$ $y = 31,$ $z = 19.$

16. $x = 4,$ $y = 6,$ $z = 8.$

17. $x = 8,$ $y = 6,$ $z = 2.$

18. $x = 1\frac{2}{3},$ $y = 4\frac{5}{6},$ $z = 7\frac{8}{9}.$

19. $x = -1.2,$ $y = 3.4,$ $z = -5.6.$

20. $x = 20,$ $y = 10,$ $z = 5.$

21. $x = 15$,
$y = 12$,
$z = 10$.

22. $x = 1$,
$y = 2$,
$z = 3$.

23. $x = \frac{2}{3}$,
$y = -7$,
$z = 36\frac{1}{3}$.

24. $x = 7$,
$y = 8$,
$z = 9$.

25. $x = 13$,
$y = 65$,
$z = 91$.

26. $x = 12$,
$y = 18$,
$z = 24$.

27. $x = 3$,
$y = 4$,
$z = 5$.

28. $x = 1$,
$y = 2$,
$z = 3$.

29. $x = 5$,
$y = 6$,
$z = 8$.

30. $x = 1$,
$y = 4$,
$z = 6$.

31. $x = 2$,
$y = 9$,
$z = 10$.

32. $x = 10$,
$y = 2$,
$z = 3$.

33. $x = 4$,
$y = 3$,
$z = 5$.

34. $x = 3$,
$y = 7$,
$z = 4$.

35. $x = 13$,
$y = 24$,
$z = 62$.

36. $x = 1$,
$y = 2$,
$z = 3$.

37. $x = 10$,
$y = 20$,
$z = 30$.

38. $x = 1$,
$y = 2$,
$z = 3$.

39. $x = 24$,
$y = 60$,
$z = 120$.

40. $x = 315$,
$y = 630$,
$z = 945$.

41. $x = 2$,
$y = 2$,
$z = 2$.

42. $x = 0$,
$y = 1$,
$z = 2$.

43. $x = 3$,
$y = 4$,
$z = 5$.

44. $x = 10$,
$y = 7$,
$z = 3$.

45. $x = 51$,
$y = 76$,
$z = 1$.

46. $x = 9$,
$y = 10$,
$z = 11$.

47. $x = 2$,
$y = 3$,
$z = 4$.

48. $x = 0$,
$y = 1$,
$z = 3$.

49. $x = 1$,
$y = 2$,
$z = 3$.

50. $x = 111$,
$y = 222$,
$z = 333$.

51. $x = 5$,
$y = 7$,
$z = 3$.

52. $x = 11$,
$y = 3$,
$z = 7$.

53. $x = 3$,
$y = 7$,
$z = 11$.

54. $x = 3$,
$y = 2$,
$z = 4$.

55. $x = \frac{4}{3}$,
$y = 4$,
$z = \frac{4}{5}$.

56. $x = 1\frac{1}{6}$,
$y = -3\frac{1}{2}$,
$z = 2\frac{1}{10}$.

57. $x = \frac{1}{2}$,
$y = \frac{2}{3}$,
$z = \frac{3}{4}$.

58. $x = \frac{1}{2}$,
$y = \frac{1}{3}$,
$z = \frac{1}{4}$.

59. $x = \frac{1}{2}$,
$y = \frac{1}{3}$,
$z = \frac{1}{4}$.

60. $x = 5$,
$y = 4$,
$z = 3$.

61. $x = 1$,
$y = 4$,
$z = \frac{1}{2}$,
$u = 1\frac{1}{2}$.

62. $x = 100$,
$y = 60$,
$z = -13$,
$u = -50$.

63. $x = \frac{1}{4}$,
$y = \frac{1}{2}$,
$z = \frac{1}{4}$,
$u = 0$.

64. $x = 12$,
$y = 5$,
$z = 7$,
$u = 4$.

65. $x=3,\quad y=1,\quad z=5,\quad u=9.$

66. $x=12,\ y=30,\ z=168,\ u=50.$

Ex. 23. Page 86.

1. $x=\frac{1}{2}(a+b-c),$
$y=\frac{1}{2}(a+c-b),$
$z=\frac{1}{2}(b+c-a).$

2. $x=\dfrac{n+p-t}{m-q},$
$y=\dfrac{mt-mp-nq}{m-q},$
$z=\dfrac{mt-nq-pq}{m-q}.$

3. $x=\dfrac{n+p}{2},$
$y=\dfrac{m-n}{2},$
$z=\dfrac{m-p}{2}.$

4. $x=-abc,$
$y=ac+ab+bc,$
$z=-(a+b+c).$

5. $x=b+c,$
$y=a+c,$
$z=a+b.$

6. $x=\dfrac{p}{2-m-n+mn},$
$y=\dfrac{p(m-1)}{2-m-n+mn},$
$z=\dfrac{p(1-m-n+mn)}{2-m-n+mn}.$

7. $x=\dfrac{amp}{mp+np+nq},$
$y=\dfrac{anp}{mp+np+nq},$
$z=\dfrac{anq}{mp+np+nq}.$

8. $x=\dfrac{2}{m+n-p},$
$y=\dfrac{2}{m-n+p},$
$z=\dfrac{2}{n+p-m}.$

9. $x=\dfrac{2a}{m+n},$
$y=\dfrac{2b}{m+p},$
$z=\dfrac{2c}{n+p}.$

10. $x=a,$
$y=b,$
$z=c.$

11. $x=\dfrac{mp-mn+np}{2mnp},$
$y=\dfrac{mn+np-mp}{2mnp},$
$z=\dfrac{mn+mp-np}{2mnp}.$

12. $x = \dfrac{2abc}{ac + bc - ab}$,

$y = \dfrac{2abc}{ab + bc - ac}$,

$z = \dfrac{2abc}{ab + ac - bc}$.

13. $x = \dfrac{abc}{ab + bc + ac}$,

$y = \dfrac{abc}{ab + bc + ac}$,

$z = \dfrac{abc}{ab + bc + ac}$.

14. $x = m^2 - n^2$,
$y = n^2 - p^2$,
$z = p^2 - m^2$.

15. $x = (m + n)(n - p)$,
$y = (n + p)(p - m)$,
$z = (p + m)(m - n)$.

16. $x = n^2 - p^2$,
$y = p^2 - m^2$,
$z = m^2 - n^2$.

17. $x = \dfrac{1}{n - p}$,

$y = \dfrac{1}{p - m}$,

$z = \dfrac{1}{m - n}$.

18. $x = n + p - m$,
$y = m + p - n$,
$z = m + n - p$.

19. $x = \frac{1}{2}(n + p)$,
$y = \frac{1}{2}(p + m)$,
$z = \frac{1}{2}(m + n)$.

20. Placing

$s = \frac{1}{3}(m + n + p + q)$

we have

$x = s - m$,
$y = s - n$,
$z = s - p$,
$u = s - q$.

21. $x = p + n$,
$y = q - p$,
$z = m + p - q$,
$u = q + n - m$,
$v = m - n$.

22. $x = m + n + p$,
$y = n - m - p$,
$z = 3m - n + p$,
$u = m + n + p$.

23. Putting

$s = \frac{1}{5}(l + m + n + p + q)$

we have

$x = s - q$,
$y = s - p$,
$z = s$,
$u = s - m$,
$v = s - n$,
$w = s - l$.

Ex. 24. Page 88.

1. The first, $3.60;
the second, $3.
2. 8 and 4.
3. A stitched copy, $3;
a bound copy, $3.80.
4. $1050, the first;
$600, the second.
5. $\frac{3}{8}$. 6. $\frac{5}{7}$.
7. 120 and 90.
8. 42 and 12.
9. 63. 12. 36.
10. 26. 13. 75.
11. 54. 14. 18 and 8.
15. 857 and 142.
16. 5 and 6.
17. 25 and 55.
18. 1.234 and 5.678.
19. 4 and $2\frac{3}{8}$.
20. Butter, 90 cts. per kilo;
soap, 75 cts. per kilo.
21. An artificer, $4;
a laborer, $2.
22. 30 mi.
23. Length, 15^m; width, 10^m.
24. Standard of the first, 0.800; standard of the second, 0.900.
25. 10 gals. of the first;
4 gals. of the second.
26. The first, $900;
the second, $400.
27. 12 men; 12 women.
28. A's library, 2400 vols.;
B's library, 1800 vols.
29. Velvet, $12.40;
silk, $1.60.
30. M, $22,000; N, $16,000;
P, $18,000.
31. First, 140 eggs;
second, 90 eggs;
third, 130 eggs.
32. Wheat, $2.80; rye,
$1.80; oats, $1.
33. 854.
34. 50, 65, and 75.
35. 640, 720, and 840.

Ex. 25. Page 93.

1. 18.05547.
2. 5.38516.
3. 22.15851.
4. 26.94438.
5. 19.77372.
6. 31.93743.
7. 30.57776.
8. 29.01723.
9. 26.81417.
10. 12.04159.
11. 3.05670.
12. 5.21458.
13. 1.91485.
14. 1.77240.
15. 1.14687.
16. 10.954.
17. 11.532.
18. 60.016.

19. 20.025.
20. 10.49.
21. 100.019.
22. 99.985.
23. 300.006.
24. 1200.003.
25. 899.997.
26. 79.975.
27. 200.007.
28. $5x-2+3y$.
29. $2x^2-3y^3+4z^5$.
30. $2-a+3a^2$.
31. $ab+a-b$.
32. x^2-4x+1.
33. $3a^2-5b+8m-z$.
34. $y^5-2u^3+3v^4-4q^5$.
35. $a^2-3c^5-4d^{2x}+3e^y$.
36. $3a^3-2a^2b-7ab^2+4b^3$.
37. $\frac{m^2}{2}-\frac{n^3}{3}+\frac{p^4}{4}$.
38. $\frac{x}{y}-\frac{2z}{u}+\frac{3q}{v}$.
39. $\frac{3}{5}-\frac{a^4}{5}-\frac{m^3}{6}+\frac{2n^4}{7}$.
40. $\frac{3m^3n^2}{5p^3q^4}-\frac{2m^2n^3}{7p^4q^5}-\frac{4mn^4}{9p^5q^6}$.
41. $\frac{3y}{4x}-5+\frac{2x}{7y}$.
42. $\frac{x^7}{a^3}-x^4-a^4$.
43. $x^3y^3-\frac{y^9}{3a^3}+\frac{2y^{15}}{a^9}$.
44. 9.535.
45. 10.746.
46. 9.920.
47. 8.025.
48. 11.785.
49. 17.002.
50. 20.779.
51. 23.368.
52. 38.356.
53. 27.550.
54. 1.481.
55. 6.824.
56. 2.488.
57. 8.836.
58. 21.551.
59. $a-3y+2b$.
60. $7x-3u+v$.
61. $5a^3-2b^2+c^4$.
62. $2x^5+3y^2-2z^3$.
63. $3a^4-b^5+6d^7$.
64. $10m^2-3n-2p$.
65. $2a^2b+3ab^2-2b^3$.
66. $5x^3y-2x^2y^3+3xy$.
67. $6m^2n^3-4m^3n^4+5mn^2$.
68. $3ax^2+2a^2x^3-5a^3x^4$.
69. $\frac{2a^2}{x}-\frac{3a^3}{x^2}+\frac{a^4}{x^3}$.
70. $\frac{m^3}{y^5}+\frac{m^2}{y^4}-\frac{m}{y^3}$.
71. $\frac{5x}{y^2}-\frac{x^2}{2y^3}+\frac{4x^3}{y^4}$.
72. $\frac{2}{a^2}+\frac{1}{a^3}-\frac{5}{a^4}$.
73. $\frac{3a}{4b}-\frac{2b}{3a^2}+\frac{5a^3}{2b^2}$.
74. $\frac{2a}{x^2}+\frac{3a^2}{x^3}+\frac{4a^3}{x^4}$.
75. $3b^2-4b^3-5b^4$.

Ex. 26. Page 97.

1. $\pm 2.$

2. $\pm \frac{\sqrt{85}}{2} = 4.6097.$

3. $\pm 5.$

4. $\pm \frac{7}{\sqrt{2}}.$

5. $\pm \sqrt{\frac{8}{5}}.$

6. $\pm 9.$

7. $\pm 4.$

8. $\pm \frac{1}{5}\sqrt{-144}.$

9. $\pm 6,$

10. $\pm \frac{2}{3}.$

11. $\pm 9.$

12. $\pm 3.$

13. $\pm \sqrt{-4}.$

14. $\pm \sqrt{-(m+n)^2}.$

15. $\pm \sqrt{a^2b^2 - 2b^2}.$

16. $\pm \sqrt{a^2b^2 - b^4}.$

17. $\pm \sqrt{a^2 + b^2 - c^2}.$

18. $\pm \sqrt{a^2 + 6ab + b^2}.$

19. $\pm 2m.$

20. $\pm \sqrt{-pn}.$

21. $\pm b.$

22. $\pm \sqrt{\frac{11a^2}{2} - 3ab + b^2}.$

23. $\pm \sqrt{(a+b)^2 + (3a)^2}$; or,
$\pm \sqrt{10a^2 + 2ab + b^2}.$

24. $\pm \sqrt{a^2 + 34ab + b^2}.$

25. $\pm (a+b).$

Ex. 27. Page 98.

1. $8;\ -6.$

2. $6;\ -14.$

3. $9;\ -19.$

4. $4;\ 3.$

5. $7;\ -2.$

6. $12;\ -5.$

7. $20;\ -19.$

8. $5;\ -2.$

9. $30;\ -21.$

10. $25;\ -10.$

11. $\frac{1}{2};\ -\frac{7}{2}.$

12. $3;\ 2.$

13. $14\frac{2}{3};\ \frac{1}{3}.$

14. $4;\ -6.$

15. $4;\ 24.$

16. $14;\ -1.$

17. $12;\ -1.$

18. $6;\ -10.$

19. $\frac{11}{2};\ -4.$

20. $1;\ -\frac{2}{5}.$

21. $3;\ -4\frac{2}{3}.$

22. $\frac{1}{3};\ -2.$

23. $\frac{6}{5};\ \frac{1}{5}.$

24. $10;\ -\frac{5}{3}.$

25. $2;\ -\frac{5}{7}.$

26. $3;\ -\frac{2}{3}.$

27. $5;\ \frac{3}{2}.$

28. $4;\ -\frac{5}{2}.$

29. $1;\ \frac{2}{3}.$

30. $\frac{3}{5};\ -\frac{33}{80}.$

31. $\frac{2}{7};\ -\frac{5}{7}.$

32. $\frac{3}{2};\ -\frac{1}{10}.$

33. $\frac{5}{6};\ -\frac{14}{15}.$

34. $2;\ -\frac{1}{9}.$

35. $2;\ -3.$

36. $5;\ \frac{2}{3}.$

37. $5;\ -\frac{10}{3}.$

38. $\frac{2}{3};\ -\frac{1}{3}.$

39. $5;\ -2\frac{1}{2}.$

40. $\frac{7}{8};\ -\frac{83}{8}.$

41. $2;\ \frac{3}{5}.$

42. $6;\ -\frac{1}{2}.$

43. $9;\ 2.$
44. $11;\ \frac{7}{2}.$
45. $1;\ -\frac{1}{16}.$
46. $\frac{3}{2};\ -\frac{15}{2}.$
47. $\frac{4}{5};\ 3\frac{1}{5}.$
48. $\frac{1}{3};\ \frac{5}{3}.$
49. $\frac{2}{3};\ -2.$
50. $1;\ \frac{3}{5}.$
51. $-2;\ \frac{24}{5}.$
52. $\frac{4}{5};\ -\frac{4}{3}.$
53. $\frac{2}{5};\ 0.$
54. $4;\ -3\frac{1}{3}.$
55. $3;\ -4\frac{5}{7}.$
56: $2;\ -\frac{3}{5}.$
57. $\frac{1}{5};\ -\frac{1}{7}.$
58. $\frac{19}{30};\ 0.3.$
59. $0.5;\ 0.1.$
60. $5;\ -7.$
61. $7;\ 1.$
62. $13;\ -\frac{13}{3}.$
63. $2;\ 40.$
64. $2.3;\ -\frac{49}{30}.$
65. $3.1;\ 0.6.$
66. $1\frac{1}{2};\ -\frac{5}{6}.$
67. $4;\ -2\frac{1}{2}.$
68. $5;\ 2\frac{1}{12}.$
69. $10;\ -0.7.$
70. $14;\ 5\frac{1}{19}.$
71. $20;\ -1\frac{1}{3}.$
72. $18;\ 15.8.$
73. $11\frac{1}{2};\ 11.$
74. $16;\ -\frac{41}{105}.$
75. $22;\ 5\frac{4}{7}.$
76. $7;\ -1.$
77. $4;\ -2.$
78. $12;\ 11.$
79. $3\frac{1}{2};\ -\frac{1}{2}.$
80. $2\frac{1}{2};\ \frac{1}{2}.$
81. $\frac{2}{3};\ -\frac{13}{21}.$
82. $6;\ -\frac{1}{3}.$
83. $11;\ 1.$
84. $17;\ 6.$
85. $6;\ -17.$
86. $2;\ -1.$
87. $5;\ 4.$
88. $7;\ -\frac{11}{3}.$
89. $1;\ -1\frac{1}{4}.$
90. $4;\ -5.$
91. $13;\ \frac{1}{2}.$
92. $7;\ 4\frac{5}{9}.$
93. $8;\ -8\frac{3}{4}.$
94. $15;\ \frac{988}{1647}.$
95. $\frac{2}{3};\ -3.$
96. $5;\ 1\frac{1}{5}.$
97. $8;\ --1.$
98. $5;\ -1\frac{1}{4}.$
99. $-7;\ -11.$
100. $-11;\ -13.$
101. $7;\ 3\frac{14}{15}.$
102. $0;\ -\frac{3}{2}.$
103. $0;\ -\frac{5}{2}.$
104. $0;\ -\frac{248}{193}.$
105. $0;\ 1.$

106. $-\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2-4b}.$
107. $\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2-4b}.$
108. $-\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2+4b}.$
109. $\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2+4b}.$
110. $\frac{a}{2}\pm\frac{1}{2}\sqrt{a^2+4b}.$
111. $-\frac{1}{2}(m\pm\sqrt{m^2-4n}).$
112. $\frac{1}{2}(d\pm\sqrt{d^2+4g}).$
113. $\frac{1}{2}(b\pm\sqrt{b^2+4c}).$
114. $4p;\ -p.$
115. $2b;\ -b.$
116. $-\frac{d}{2}\pm 2k^2.$

117. $\frac{3}{2a}$; $-\frac{1}{2a}$.

118. $-m$; $-n$.

119. n; p.

120. $\frac{m}{n}$; $\frac{n}{m}$.

121. $\frac{3n}{4}$; $-2\frac{1}{4}n$.

122. c; 2.

123. a; $-\frac{a}{2}$.

124. $\frac{2m}{3}$; $-m$.

125. $\frac{p}{2}$; $\frac{p}{10}$.

126. $\frac{3b}{2}$; $\frac{b}{6}$.

127. $\frac{3(a+b)}{2}$; $-\frac{a+b}{2}$.

128. m; $-n$.

129. a; $-b$.

130. $m+n$; $2n$.

131. $-a$; $-b$.

132. $-\frac{1}{6}(a \pm \sqrt{a^2+12b})$.

133. $2a$; $-\frac{a}{2}$.

134. $\frac{1}{2b}(a \pm \sqrt{a^2-4bp})$.

135. 1; $-\frac{5a+c}{c}$.

136. $-\frac{a}{2}$; $-\frac{9a}{2}$.

137. $\frac{a}{2}$; $\frac{b}{2}$.

138. $2p$; $\frac{p}{2}$.

139. $\frac{ab}{2a-b}$; $2b$.

140. $4a-3b$; $3b-a$.

141. $\frac{a+3b}{2}$; $-\frac{3a+b}{2}$.

142. $\frac{3p}{4}$; $-p$.

143. $\frac{b}{a}$; $-\frac{d}{c}$.

144. $\frac{2a-b}{ac}$; $-\frac{3a+2b}{bc}$.

145. $\frac{a}{a-b}$; $\frac{b}{a-b}$.

146. $\frac{1}{ab^2}$; $\frac{1}{a^2b}$.

147. $-\frac{3a}{2}$; $-4a$.

148. $\frac{b}{a} \pm \frac{1}{a}\sqrt{b^2+ac}$.

149. $-\frac{b}{a} \pm \frac{1}{a}\sqrt{b^2-ac}$.

150. $\frac{2q}{p} \pm \frac{1}{p}\sqrt{4q^2+mp}$.

151. $-2 \pm \frac{1}{m}\sqrt{4m^2-cm}$.

152. $\frac{8+a}{c}$; $-\frac{a}{c}$.

153. $5a$; $\frac{7a}{3}$.

154. $\frac{7ab}{3}$; $\frac{3ab}{5}$.

155. $ab \pm a\sqrt{b^2 - b}$.

156. $\frac{2m}{3}$; $-\frac{2n}{3}$.

157. $\frac{a+b-ab}{b+1}$; $-\frac{a+b+ab}{b+1}$.

158. $-\frac{a-b}{3}$; $-(a-b)$.

159. $a-b$; $-(a+3b)$.

160. $\frac{b^3}{ac}$; $\frac{b^2}{ac}$.

161. $a+2b$; $a-2b$.

162. $a^2+b^2 \pm \sqrt{a^4+b^4}$.

163. $-c \pm \sqrt{c^2+ab-bc-ac}$.

164. $-\frac{a}{2}(3 \pm \sqrt{3})$.

165. $\frac{a}{2}(3 \pm \sqrt{-3})$.

166. $\frac{2a}{3}$.

167. $\frac{ab}{c}$; $-\frac{c^3}{ab}$.

168. $\frac{a}{b}$; $-\frac{b}{a}$.

169. $3a-2b$; $2a+3b$.

170. $2b+a$; $2b-a$.

171. $a+2c$; $b-c$.

172. $a+b-2c$; $b+2c$.

173. a^2+b^2; $2ab$.

174. $\frac{3b}{c}$; $-\frac{5d}{a}$.

175. $\frac{a^3b}{c^4}$; $-\frac{b^2c^3}{a^5}$.

176. $\frac{2n^4}{m^6p^2}$; $-\frac{2n^2}{m^3p^5}$.

177. a^2-b^2; $a+b$.

178. $2m(2m-n)$; $n(2m+n)$.

179. $\frac{2ab-ac-bc}{a+b-2c}$; 0.

180. c; $-\frac{d(c+d)}{2c+d}$.

181. $2p$; 0.

182. 0; $\frac{bp-cn}{ap-cm}$.

Ex. 28. Page 107.

1. ± 35; ± 21.

2. ± 19; ± 20; ± 21.

3. ± 14; ± 16; ± 18.

4. ± 28; ± 32; ± 36.

5. $229,376.

6. ± 59; ± 47.

7. $(12\sqrt{3})^m$; $(6\sqrt{3})^m$.

8. 96; 66.

9. 5 dys.

10. 3^{kg}.

11. 5.996 dys.

12. 10, 11, 12, 13, 14; or $-22, -23, -24, -25, -26$.
13. 18; or -22.
14. 72, 24; or 1, 49.
15. ± 4080.
16. 81, 27; or $-80, -\frac{80}{3}$.
17. 21; or -22.
18. 36 yds.; 24 yds.
19. 391^{m}; 80^{m}.
20. $13\frac{1}{3}$ cts.
21. 20 books.
22. 21 cattle; $32.
23. 225; 441.
24. 36 yrs.; 12 yrs.
25. 18 dys.; 35 mi.
26. 14 dys.; 45 mi.
27. 42 melons.
28. 28 children.
29. $65; or $35.
30. 276 yds.; 190 yds.
31. 130 yds.
32. 2 yds.
33. 4 boys; 3 girls.
34. 12^{dm}; 5^{dm}.
35. 6 dys.
36. 12 oranges.
37. $\frac{4}{7}$.
38. 20^{1}.
39. First, 9 mi.; second, 8 mi.
40. First, at 8 o'clock; second, at 7 o'clock, in the evening.
41. Larger, 45 hrs.; smaller, 63 hrs.
42. $\pm m\sqrt{\frac{s}{m^2+n^2}}$; $\pm n\sqrt{\frac{s}{m^2+n^2}}$.
43. $x = b + c - a \pm \sqrt{2(a-b)(a-c)}$; $x = 1$.
44. $\frac{a}{2}$.
45. 17 friends.
46. ± 640.
47. 196; $(-13)^2$.
48. 14 hrs.; $17\frac{1}{2}$ hrs.
49. 32 sec.; or 34.682 sec.

Ex. 29. Page 114.

1. $x^2 - 8x + 15 = 0$.
2. $x^2 - 10x + 21 = 0$.
3. $x^2 - 3x + 2 = 0$.
4. $x^2 - 25x + 136 = 0$.
5. $x^2 - 2x - 3 = 0$.
6. $x^2 - 5x - 14 = 0$.
7. $x^2 + x - 30 = 0$.
8. $x^2 + 5x - 36 = 0$.

9. $x^2 - \frac{7}{6}x + \frac{1}{3} = 0$; or $6x^2 - 7x + 2 = 0$.

10. $x^2 - \frac{19}{12}x + \frac{5}{8} = 0$; or $24x^2 - 38x + 15 = 0$.

11. $x^2 - 12\frac{7}{8}x + 10\frac{1}{2} = 0$.

12. $x^2 - 3\frac{3}{5}x - \frac{8}{5} = 0$.

13. $x^2 + 6\frac{5}{8}x - \frac{21}{8} = 0$.

14. $x^2 + 2\frac{1}{5}x + \frac{2}{5} = 0$.

15. $x^2 - 1 = 0$.

16. $x^2 - 121 = 0$.

17. $x^2 + 10x + 24 = 0$.

18. $x^2 + 12x + 20 = 0$.

19. $x^2 + 16x + 55 = 0$.

20. $x^2 + 3\frac{1}{4}x + \frac{3}{4} = 0$.

21. $x^2 - 6.8x + 10.75 = 0$.

22. $x^2 - 2.9x + 1 = 0$.

23. $x^2 + 0.4x - 0.21 = 0$.

24. $x^2 + 3.1x - 41.82 = 0$.

25. $x^2 + 3.7x + 3.12 = 0$.

26. $x^2 - 2.5x + 1 = 0$.

27. $x^2 - 4.25x + 1 = 0$.

28. $x^2 + 7\frac{1}{7}x + 1 = 0$.

29. $x^2 - 4\frac{4}{5}x - 1 = 0$.

30. $x^2 + 2\frac{2}{3}x - 1 = 0$.

31. $x^2 - (a+b)x + ab = 0$.

32. $x^2 - 2ax + a^2 - b^2 = 0$.

33. $x^2 - a(1+a)x + a^3 = 0$.

34. $x^2 - 4(a+b)x + 3(a^2+b^2) + 10ab = 0$.

35. $x^2 - a^2 = 0$.

36. $x^2 - (a-b)x - ab = 0$.

37. $x^2 + (m+n)x + mn = 0$.

38. $x^2 - mx - 6m^2 = 0$.

39. $x^2 - (2+m)x + 2m = 0$.

40. $6x^2 - (3a+2)x + a = 0$.

41. When $p^2 \geqq 4q$; or when $q < 0$.

42. When $p^2 < 4q$.

43. When $p^2 > 4q$.

44. When $p^2 = 4q$ and q positive.

45. When p is negative and $> \sqrt{p^2 - 4q}$.

46. When p is positive and $> \sqrt{p^2 - 4q}$.

47. When p is negative and $< \sqrt{p^2 - 4q}$.

48. When p is positive and $< \sqrt{p^2 - 4q}$.

49. When $p^2 - 4q$ is a perfect square.

50. When q is negative, the roots are always real. If $p^2 - 4q$ is not a perfect square, the roots are irrational. If the equation has the form $ax^2 + bx + c = 0$, p must be replaced by $\frac{b}{a}$, and q by $\frac{c}{a}$.

51. Real and positive (4 ; 2).
52. Real and positive (3 ; 2).
53. Real with opposite signs (4 ; -3).
54. Real and negative (-1 ; -11).
55. Real and negative (-1 ; -8).
56. Real and negative (-1 ; -14).
57. Real $\left(\frac{3}{2}; -\frac{29}{2}\right)$.
58. Real (1 ; -26).
59. Real (1 ; -12).
60. Real (10 ; -2).
61. Real (23 ; -2).
62. Real (7 ; 9).
63. Real and equal (8).
64. Real and equal (9).
65. Real and equal (-5).
66. Imaginary $\left(\frac{9 \pm \sqrt{-49}}{2}\right)$.
67. Imaginary $[\frac{1}{2}(31 \pm \sqrt{-25})]$.
68. Irrational $(1 \pm \sqrt{6})$.
69. Irrational $(-7 \pm \sqrt{51})$.
70. Irrational $(-3 \pm \sqrt{5})$.
71. Imaginary $(-4 \pm \sqrt{-9})$.
72. $cx^2 + bx + a = 0$.

Ex. 30. Page 116.

1. Max. $= 16$ for $x = 0$.
2. Max. $= 9$ for $x = 0$.
3. Max. $= 16$ for $x = 4$.
4. Min. $= -25$ for $x = 5$.
5. Min. $= -4$ for $x = 3$.
6. Min. $= -4$ for $x = -\frac{1}{4}$.
7. Min. $= -4$ for $x = \frac{2}{3}$.
8. Min. $= -3$ for $x = 1$.
9. Min. $= -15$ for $x = 2$.
10. Max. $= 8$ for $x = 1$.
11. Min. $= 14$ for $x = 2$.
12. Max. $= 4$ for $x = 2$.
13. Max. $= \frac{1}{24}$ for $x = 12$
14. Max. $= \frac{1}{4}$ for $x = 2$.
15. Max. $= \frac{25}{16}$ for $x = 8$.
16. Max. $= \infty$ for lim. $x = \pm 3$.
Min. $= 2$ for $x = 0$.

17. Max. $= \infty$ for lim. $x = \pm 2$.
Min. $= 1$ for $x = 0$.

18. Max. $= 1$ for $x = 1$.
Min. $= -1$ for $x = -1$.

19. Max. $= 2$ for $x = 3$.
Min. $= -1$ for $x = 0$.

20. Max. $= \frac{1}{2}$ for $x = 2$.

21. Min. $= \dfrac{2mn}{m^2 - n^2}$ for $x = \dfrac{n}{m}$.

22. Min. $= 3\frac{3}{7}$ for $x = \frac{3}{4}$.

23. Max. $= 4$ for $x = 1$.
Min. $= 0$ for $x = -1$.

24. Min. $= 0$ for $x = 1$.

25. Min. $= -\frac{5}{8}$ for $x = \frac{1}{2}$.

26. Min. $= 10$ for $x = 5$.
Max. $= -10$ for $x = -5$.

27. Min. $= 8$ for $x = 4$.
Max. $= -8$ for $x = -4$.

28. Min. $= 45$ for $x = 10$.
Max. $= 5$ for $x = -10$.

29. Min. $= 28$ for $x = 6\frac{1}{4}$.
Max. $= 2$ for $x = -\frac{1}{4}$.

30. Max. $= 3$ for $x = 2$.
Min. $= \frac{3}{2}$ for $x = -1$.

31. Max. $= 1$ for $x = -\frac{1}{2}$.
Min. $= -\frac{1}{5}$ for $x = -\frac{7}{2}$.

32. Max. $= -3$ for $x = 1$.
Min. $= -\frac{1}{3}$ for $x = -1$.

33. Max. $= 5.5$ for $x = \frac{1}{3}$.
Min. $= 0.5$ for $x = -3$.

34. Max. $= \frac{1}{7}$ for $x = 5$.
Min. $= \frac{1}{3}$ for $x = 3$.

35. Max. $= \dfrac{1}{2a}$ for $x = a$.
Min. $= -\dfrac{1}{2a}$ for $x = -a$.

36. Min. $= 4$ for $x = 1$.
Max. $= \infty$ for lim. $x = 0$.

37. Min. $= 4$ for $x = 2$.
Max. $= \infty$ for lim. $x = 1$.

38. Min. $= 8$ for $x = 5$.
Max. $= \infty$ for lim. $x = 3$.

39. Min. $= 1$ for $x = 2$.
Max. $= \infty$ for lim. $x = 1$.

40. Min. $= -(5 + 2\sqrt{6})$ for $x = \sqrt{6}$.

41. Min. $= -3$ for $x = 0$.
Max. $= +\infty$ for $x = -(1 + h)$.
Min. $= -\infty$ for $x = -(1 - h)$.

42. Min. $= 20\frac{22}{27}$ for $x = \frac{8}{3}$.
Max. $= 30$ for $x = -2$.

43. Max. $= 3.1219$ for $x = \frac{2-\sqrt{7}}{3}$.
Min. $= 0.369$ for $x = \frac{2+\sqrt{7}}{3}$.

44. Max. $= 19\frac{19}{37}$ for $x = -\frac{1}{3}$.
Min. $= 6$ for $x = 2$.

45. Max. $= -20\frac{1}{6}$ for $x = 1$.
Min. $= -20\frac{1}{3}$ for $x = 2$.

46.

When x increases from	y varies from
$-\infty$ to $+1$,	$+\infty$ to 0,
1 to 3,	0 to -4 (min.),
3 to 5,	-4 to 0,
5 to $+\infty$,	0 to $+\infty$.

47.

When x varies from	y varies from
$-\infty$ to -3,	$+\infty$ to 0,
-3 to -2,	0 to -25 (min.),
-2 to 0,	-25 to -9 (max.),
0 to 2,	-9 to -25 (min.),
2 to 3,	-25 to 0,
3 to $+\infty$,	0 to $+\infty$.

48.

When x varies from	y varies from
$-\infty$ to -1,	$-\infty$ to -2 (max.),
-1 to $-h$,	-2 to $-\infty$,
$-h$ to $+h$,	$-\infty$ to $+\infty$,
$+h$ to 1,	$+\infty$ to 2 (min.),
1 to ∞,	2 to ∞.

49.

When x varies from	y varies from
$-\infty$ to 1,	$+\infty$ to -2 (min.),
1 to 2,	-2 to $+\infty$,
2 to 3,	$+\infty$ to -6 (max.),
3 to ∞,	-6 to $-\infty$.

50. When x varies from	y varies from
$-\infty$ to -1,	1 to $\frac{7}{8}$ (max.),
-1 to 0,	$\frac{7}{8}$ to 1,
0 to $+1$,	1 to -3 (min.),
$+1$ to $+\infty$,	-3 to 1.

51. When x varies from	y varies from
$-\infty$ to -7,	0 to $\frac{1}{14}$ (max.),
-7 to -3,	$\frac{1}{14}$ to $-\frac{1}{12}$ (min.),
-3 to -1,	$-\frac{1}{12}$ to $\frac{1}{2}$ (max.),
-1 to $+2$,	$\frac{1}{2}$ to 2,
2 to 5,	2 to $-\frac{1}{2}$.

52. Let x be one part; $2a-x=$ the other; then $(2a-x)x=m$; whence $x=a$. The parts should be equal.

53. A square. 54. Each part $=a$. 55. Each part $=a$.

56. Let a, b, c be the sides of a triangle, and $2p$ be its perimeter. The surface is $\sqrt{p(p-a)(p-b)(p-c)}=m$. Now p is constant; in order that there may be a maximum, it is necessary that $p-a=p-b=p-c$; that is to say, $a=b=c$.

57. Let a and b be the sides of a rectangle; r the radius of the circle. The area of the rectangle is $ab=m$. But $a^2+b^2=4r^2$, or $a^2+\frac{m^2}{a^2}=4r^2$; whence $a^2=2r^2\pm\sqrt{4r^4-m^2}$. The radical must be equal to zero, which makes $a=r\sqrt{2}$, and $m=r\sqrt{2}\times r\sqrt{2}=2r^2$. The figure is then the inscribed square.

58. Let a be the base, and h the height of the triangle; x the base, and y the height of the rectangle. Then the surface $xy=m$. But $\frac{h}{h-y}=\frac{a}{x}$; whence $x=\frac{a(h-y)}{h}$, and $xy=\frac{(ah-ay)y}{h}=m$; or $ay^2-ahy=-mh$; whence $y=\frac{h}{2}\pm\sqrt{\frac{h^2}{4}-\frac{mh}{a}}$. If the radical $=0$, $y=\frac{h}{2}$, $x=\frac{a}{2}$.

59. Let x be the radius of the cylinder, and y its height. Then $\frac{h}{h-y}=\frac{r}{x}$; whence $x=\frac{r(h-y)}{h}$. The volume of the cylinder is $\pi x^2 y=\frac{\pi r^2}{h^2}(h-y)^2 y$. The maximum depends solely upon the factors $(h-y)^2$ and y, the sum of whose first powers is constant. Then we must have for the product, as maximum, $\frac{h-y}{2}=\frac{y}{1}$; whence $y=\frac{h}{3}$, and $x=\frac{2r}{3}$.

60. Start from each angle of the given square, and, going in the same direction, take the distances equal x, and join two and two the consecutive points thus obtained. The figure inscribed will be a square. Let h be the side of this square, of which the area $h^2=M$. But $h^2=x^2+(a-x)^2=2x^2-2ax+a^2=M$. Then $x=\frac{a}{2}\pm\frac{1}{2}\sqrt{2M-a^2}$. Then, $M=\frac{a^2}{2}$, $x=\frac{a}{2}$, and $h=\frac{a\sqrt{2}}{2}$.

61. Let R be the radius of the base of the cone, and H be its height. The volume, to be a minimum, is $\frac{\pi R^2 H}{3}$. But $\frac{H}{H-h}=\frac{R}{r}$; whence $R=\frac{Hr}{H-h}$; then $\frac{\pi R^2 H}{3}=\frac{\pi H^3 r^2}{3(H-h)^2}=\frac{\pi r^2}{3}\times\frac{H^3}{(H-h)^2}$. The minimum depends only on $\frac{H^3}{(H-h)^2}$, which can be written

$$\frac{H^3}{H^2-2Hh+h^2}=\frac{1}{\frac{1}{H}\left(1-\frac{h}{H}\right)^2}=\frac{h}{\frac{h}{H}\left(1-\frac{h}{H}\right)^2}.$$

The first powers of the two factors of the denominator give a constant sum; their product is a maximum when we have $\frac{\frac{h}{H}}{1}=\frac{1-\frac{h}{H}}{2}$; whence $H=3h$, and $R=\frac{3r}{2}$.

62. Let R be the radius of the sphere; r the radius of the base of the cylinder; h its height. Then $\frac{h^2}{4}+r^2=R^2$; then $r^2=R^2-\frac{h^2}{4}$. The surface of the cylinder $=2\pi rh$; but 2π is constant, then $rh=\text{max.}=m$; whence $r^2=\frac{m^2}{h^2}$, and $R^2-\frac{h^2}{4}=\frac{m^2}{h^2}$. Therefore, $h^2=2R^2\pm2\sqrt{R^4-m^2}$. Then R^4 must $=m^2$; then $m=R^2$; $h=R\sqrt{2}$, and $r=\frac{R\sqrt{2}}{2}$. The height and the diameter are equal to a side of the square inscribed.

63. Let $2x$ be the base of the triangle; h its height; R the radius of the circle circumscribed. Surface $xh=m$. But $x^2+(h-R)^2=R^2$; then $x=\sqrt{2hR-h^2}$. Then $h^2(2hR-h^2)=m^2=h^3(2R-h)$. The sum $2R-h+h$ being constant, we have $\frac{h}{3}=\frac{2R-h}{1}$; whence $h=\frac{3R}{2}$, and $x=\frac{R\sqrt{3}}{2}$; the base is then $R\sqrt{3}=$ side of the equilateral triangle.

64. Let R be the radius of the sphere; r the radius of the base of the cone; h the height of the cone; D the distance from the summit to the circle of contact with the sphere. We have $\frac{r}{R}=\frac{h}{D}$. But $D^2=(h-R)^2-R^2$; then $\frac{r}{R}=\frac{h}{\sqrt{(h-R)^2-R^2}}$; whence $r^2=\frac{R^2h}{h-2R}$. The volume of the cone is $\frac{\pi r^2h}{3}=\frac{\pi}{3}\times\frac{R^2h^2}{h-2R}=m$; as $\frac{\pi}{3}$ is constant it is suppressed, and

$$h=\frac{m}{2R^2}\pm\sqrt{\frac{m^2}{4R^4}-\frac{8mR^3}{4R^4}}.$$

Making the radical equal to zero, we have $m=8R^3$; $h=4R$; and $r=R\sqrt{2}$.

65. Let R be the radius of the sphere; r the radius of the base of the cone; h its height; A its apothem. The surface, to render maximum, is $\pi r A$, or rA, π being constant. Now $r^2=(R+x)(R-x)$; $A^2=2R(R+x)$; whence $r^2A^2=2R(R+x^2)^2(R-x)$. Now $2R$ is constant, and the sum of the factors $R+x$ and $R-x$ is constant; then $\frac{R+x}{2}=R-x$; then $x=\frac{R}{3}$, and $h=\frac{4R}{3}$.

Ex. 31. Page 119.

1. $x=\pm 25$; $y=\pm 6$.

2. $y=\pm 1$; $z=\pm 9$.

3. $x=\pm 5$; $y=\pm 4$.

4. $x=\pm 8$; $y=\pm 6$.

5. $x=7,\ 5$; $y=5,\ 7$.

6. $y=20,\ -12$; $z=12,\ -20$.

7. $x=\frac{2}{3},\ \frac{1}{2}$; $y=\frac{1}{2},\ \frac{2}{3}$.

8. $x=\frac{1+\sqrt{5}}{2},\ \frac{1-\sqrt{5}}{2}$; $y=\frac{1-\sqrt{5}}{2},\ \frac{1+\sqrt{5}}{2}$.

9. $x=7,\ 5$; $y=5,\ 7$.

10. $x=13,\ -13$; $y=11,\ -11$.

11. $x=9,-12$; $y=12,\ -9$.

12. $x=7,\ -3\frac{100}{133}$; $y=8,\ 12\frac{40}{133}$.

13. $x=3,\ -\frac{33}{8}$; $y=4,\ -\frac{11}{2}$.

14. $x=5,\ 2,\ 6,\ 1$; $y=2,\ 5,\ 1,\ 6$.

15. $y=11,\ 3$; $z=3,\ 11$.

16. $x=\frac{3}{4},\ \frac{1}{5}$; $z=\frac{1}{5},\ \frac{3}{4}$.

17. $x=13,\ 9$; $y=-9,\ -13$.

18. $x=13$; $y=7$.

19. $x=\frac{2}{3}$; $y=\frac{5}{6}$.

20. $x=13,\ -13$; $y=7,\ -7$.

21. $x=\pm 13,\ \pm 10\sqrt{2}$; $y=\pm 7,\ \pm 3\sqrt{2}$.

22. $x=7,\ -\frac{6}{5}$; $y=-3,\ 17\frac{1}{2}$.

23. $x=\pm 3$; $y=\pm 1$.

24. $x = 5, 14\frac{51}{71}$;
$y = 2, 15\frac{48}{71}$.

25. $x = 7, 5$;
$y = 5, 7$.

26. $x = 9, 8$;
$y = 8, 9$.

27. $x = 12$;
$y = 11$.

28. $x = 5, 4$;
$y = 3, 4$.

29. $x = 8, 13$;
$y = 12, 7$.

30. $x = 7, 4\frac{9}{11}$;
$y = 2, 3\frac{5}{11}$.

31. $x = 4, 5$;
$y = 5, 4$.

32. $x = 6, 3$;
$y = 3, 6$.

33. $x = 5, -12$;
$y = 12, -5$.

34. $x = \pm 7, \pm 3$;
$y = \pm 3, \pm 7$.

35. $x = \pm 11, \pm 6$;
$y = \pm 6, \pm 11$.

36. $x = \pm 14, \pm 12$;
$y = \pm 12, \pm 14$.

37. $x = 7, 2$;
$y = 2, 7$.

38. $x = 4, \frac{1}{2}$;
$y = \frac{1}{2}, 4$.

39. $x = 7, -\frac{2}{3}$;
$y = \frac{2}{3}, -7$.

40. $x = 10, -8$;
$y = 8, -10$.

41. $x = 5, -6$;
$y = 11, -12$.

42. $x = \frac{3}{4}, -\frac{7}{4}$;
$y = \frac{1}{4}, -\frac{5}{4}$.

43. $x = 3, 2, -3 \pm \sqrt{3}$;
$y = 2, 3, -3 \pm \sqrt{3}$.

44. $x = 4, 1$;
$y = 1, 4$.

45. $x = 11, -3$;
$y = 3, -11$.

46. $x = 5, 3$;
$y = 3, 5$.

47. $x = 7, -4$;
$y = 4, -7$.

48. $x = 6, 4$;
$y = 4, 6$.

49. $x = \pm 2, \pm\frac{58}{19}$;
$y = \pm 3, \pm\frac{47}{19}$.

50. $x = 5, 3$;
$y = 3, 5$.

51. $x = 7, -2$;
$y = 2, -7$.

52. $x = 8, 7$;
$y = 7, 8$.

53. $x = 11, -9$;
$y = 9, -11$.

54. $y = 1, 4$;
$x = 4, \frac{1}{16}$.

55. $x = 6, 5$;
$y = 5, 6$.

56. $x = 5, 3$;
$y = 3, 5$.

57. $x = 8, 7$;
$y = 7, 8$.

58. $x = 4, 3$;
$y = 3, 4$.

59. $x = 3$;
$y = 1$.

60. $x = 6, \frac{41}{4}$;
$y = 4, \frac{39}{4}$.

61. $x = 6, -2$;
$y = -2, 6$.

62. $x = \pm 12$;
$y = \pm 6$;
$z = \pm 9$.

63. $x = \pm 9$;
$y = \pm 7$;
$z = \pm 3$.

64. $x = 8, -16$;
$y = 4, -2$;
$z = 6, \frac{6}{7}$.

65. $x = \pm 6$;
$y = \pm 8$;
$z = \pm \frac{1}{2}$.

66. $x = \frac{8}{3}, \frac{3}{2}$;
$y = \frac{3}{2}, \frac{8}{3}$;
$z = 2, 2$.

67. $x = 20, -20$;
$y = 4, -4$.

71. $x = 5, \frac{50}{17}$;
$y = 1, \frac{52}{17}$;
$z = 6, \frac{123}{17}$;
$u = 4, \frac{47}{17}$.

68. $x = 11, -20$;
$y = 8, -\frac{22}{5}$.

69. $x = 7, -22$;
$y = 8, 37$.

70. $x = 2, -7$;
$y = 5, 4$;
$z = 4, 5$;
$u = 3, 12$.

Ex. 32. Page 122.

1. 128 sheep; \$2.50 per head.

2. 50 sheep; \$6.40 per head.

3. 42 cabbages, at \$1.20 a doz.; or 60 cabbages, at \$0.84 a doz.

4. 48 melons, at $12\frac{1}{2}$ cents apiece; or 40 melons, at 15 cents apiece.

5. Blue, 60 yds., at \$6, or $24\frac{6}{11}$ yds., at \$$14\frac{2}{3}$; black, 30 yds., at \$16, or $65\frac{5}{11}$ yds., at \$$7\frac{1}{3}$.

6. Front wheel, 2.5^{m}; back wheel, 3.9^{m}.

7. 1200 men; ration, 1^{kg}.

8. 1800 men; ration, $2\frac{1}{2}$ lbs.

9. 28 workmen and 24^{kg}; or 36 workmen and 56^{kg}.

10. Carriage, 54 hours; train, 27 hours.

150^{km}.

12. $x = 80^{m}$; $y = 39^{m}$.

13. Length, 63 ft.; width, 16 ft.

14. The greater, 28 ft.; the smaller, 21 ft.

15. 36 yds. and 29 yds.

16. 12 ft. and 36 ft.

17. Watch, \$18; cane, \$6; umbrella, \$12; ring, \$9.

18. A, \$58; B, \$44; C, \$62; D, \$34; or
A, \$44; B, \$58; C, \$34; D, \$62.

19. 8473.

20. $x = \pm\frac{b+1}{2}\sqrt{\frac{a}{b}}$; $y = \pm\frac{b-1}{2}\sqrt{\frac{a}{b}}$.

Ex. 33. Page 126.

1. $x = 11, 10, \ldots\ldots;$
$y = 1, 2, \ldots\ldots$

2. $x = 1, 3, 5, 7;$
$y = 36, 25, 14, 3.$

3. $x = 9;$
$y = 3.$

4. $x = 2;$
$y = 1.$

5. $x = 9, 1;$
$y = 3, 6.$

6. $x = 50, 37, 24, 11;$
$y = 1, 9, 17, 25.$

7. $x = 1, 4, 7, \ldots\ldots;$
$y = 142, 137, 132, \ldots\ldots$

8. $x = 54, 45, 36, \ldots\ldots, 9;$
$y = 8, 16, 24, \ldots\ldots, 48.$

9. $x = 12, 3;$
$y = 3, 9.$

10. $x = 11, 2;$
$y = 1, 5.$

11. $x = 116, 96, \ldots\ldots, 16;$
$y = 1, 4, \ldots\ldots, 16.$

12. $x = 35, 34, 33, \ldots\ldots, 1;$
$y = 3, 6, 9, \ldots\ldots, 105.$

13. $x = 18, 15, 12, \ldots\ldots, 3;$
$y = 5, 10, 15, \ldots\ldots, 30.$

14. $x = 22, 21, 20, \ldots\ldots, 1;$
$y = 1, 3, 5, \ldots\ldots, 43.$

15. $x = 5, 10, 15;$
$y = 21, 14. 7.$

16. $x = 13;$
$y = 8.$

17. $x = 2;$
$y = 10.$

18. $x = 10, 3;$
$y = 8, 12.$

19. $x = 22, 69, 116, \ldots\ldots;$
$y = 7, 22, 37, \ldots\ldots$

20. $x = 16, 39, 62, \ldots\ldots;$
$y = 65, 159, 253, \ldots\ldots$

21. $x = 150, 133, 116, 99, 82, 65, 48, 31, 14$;
$y = 5, 12, 19, 26, 33, 40, 47, 54, 61$.

22. $x = 14, 31, 48, 65, 82$;
$y = 6, 14, 22, 30, 38$.

23. $x = 6, 21, 36, 51, 66$;
$y = 2, 9, 16, 23, 30$.

24. $x = 4, 24, 44, 64$, ...;
$y = 10, 30, 50, 70$, ...

25. $x = 81, 57, 33, 9$;
$y = 8, 32, 56, 80$.

26. $x = 264, 121$;
$y = 52, 195$.

27. $x = 63$;
$y = 55$.

28. 39.

29. 15, 71, 127, 183, 239, ...

30. First number, 213, 301, 389, ...;
second number, 62, 150, 238, ...

31. First part, 303, 240, 177, 114, 51;
second part, 49, 112, 175, 238, 301.

32. $x = 16, 13, 10, 7, 4, 1$;
$y = 2, 4, 6, 8, 10, 12$.

33. $x = 8, 19, 30, 41$, ...;
$y = 2, 15, 28, 41$, ...

34. $5 bills, 1, 3, 5, 7, 9, 11, 13, 15, 17;
$2 bills, 41, 36, 31, 26, 21, 16, 11, 6, 1.

35. Calves, 3, 6, 9, 12, 15, 18, 21;
pigs, 33, 28, 23, 18, 13, 8, 3.

36. Lambs, 3, 10; geese, 39, 10.

37. Five-franc pieces, 9; one-franc pieces, 29.

38. Sovereigns, 20, 40, 60, ...;
twenty-centime pieces, 936, 3457, 5978, ...

39. Weights of 30^g, 1, 2, 3, 4, 5, ...;
weights of $2\frac{1}{2}^g$, 388, 376, 364, 352, 340, ...

40. 11 grown persons; 6 children.

41. 334 pears.

42. 41, 34, 27, 20, 13, 6 yds. of silk;
1, 6, 11, 16, 21, 26 yds. of velvet.

43. First, \$5; second, \$4.

44. 41 peaches; 8 doz. apples.

45. 22 yrs. (born in 1858).

46. Born in 1786, and 22 yrs. old.

47. Potatoes, 1, 1, 2, 3, 4, 5, 6, 8;
apples, 2, 6, 3, 4, 1, 2, 3, 1;
squashes, 30, 5, 20, 10, 25, 15, 5, 10.

48. The number can be expressed in three ways: $2x+1$; $3y+2$; $7z+5$; whence $2x+1=3y+2=7z+5$, which gives $x=23, 44, 65, 86, \ldots\ldots;$

$y=15, 29, 43, 57, \ldots\ldots;$

$z=6, 12, 18, 24, \ldots\ldots$

The simultaneous values of the three unknown quantities give the same value for the numbers 47, 89, 131, 173, 215, ...

49. 503, 1007, 1511, 2015, ...

50. Calves, \$7; lambs, \$4; pigs, \$3.

51. 10, 45, 56.

Ex. 34. Page 129.

1. $13>5$.
2. $2a-4<x^2+2x$.
3. $x^2+y^2+y-2a>7-a+b$.
4. $x+4y<13$.
5. $12>5$.
6. $4<9$.
7. $a^2-2x<3-x-b^2$.
8. $ax^2>b^2-8$.
9. $21>-7$.
10. $-6<-1$.
11. $-8a<2x$.
12. $-(a-b)^2<-(x+y)^2$.
13. $18a^2-a^2y^3<5a^2$.
14. $3>2$.
15. $21>13$.
16. $a+b>1$.
17. $-(x^2+2b)<-7x^2$.
18. $a^2-1<70$.
19. $a^2-b^2>mn$.
20. $a^3-b^3>x^2-y^2$.

21. $5\frac{1}{3} > \frac{2}{7}$.
22. $x - y > \frac{1}{2}a$.
23. $900 > 144$.
24. $(a+b)^3 > (a-x)^3$.
25. $(x+1)^4 < y^4$.
26. $a^5 < (b-1)^5$.
27. $-1 > -32$.
28. $(3-e)^7 > -1$.
29. $(1+a)^4 > b^4$.
30. $-a^3(1+m)^3 > x^3$.
31. $3 > 2$.
32. $-5 < +4$.
33. $9 > 7$.
34. $6 > 5$, or $-6 < -5$.

35. $4 + ab > 2x - (a^2 + b^2)$.
36. $-(2x^2+1) < x$.
37. $x < y$.
38. $x > -\frac{1}{2}$.
39. $x > 6$.
40. $x > \frac{24}{25}$.
41. $x < 3\frac{11}{15}$.
42. $x < \frac{11}{12}$.
43. $x > -\frac{8m}{7}$.
44. $x > \frac{25}{18}$.
45. $x > \frac{23}{43}$.
46. $x < -2\frac{13}{20}$.
47. $x > \frac{20}{7}$; $y > 12 - 4x$, [$< \frac{2x-4}{3}$.

48. $x < -\frac{2}{5}$; $y > \frac{3x}{2}$, $< \frac{2x-1}{3}$.
49. $x < -13$; $y > \frac{10x+1}{3}$, $< 3x - 4$.
50. $x < -\frac{7}{6}$; $y > 7x + 4$, $< x - 3$.
51. $x > \frac{16}{3}$; $y < -(x+4)$, $> \frac{8-5x}{2}$.

55. Always.
56. Always.
57. Always.
58. $x > 2$, or < -3.

60. $x > -5$ and < 3.
61. Impossible.
62. Impossible.
63. $x < 4$ and > -20.

59. x included between $-\frac{1}{2}$ and $-\frac{2}{3}$.
64. x included between 7 and -3.
65. x included between 6 and -2.
66. x included between $\frac{5}{2}$ and -3.
67. $x < \frac{1}{3}$, or $> \frac{5}{2}$. **68.** $x > \frac{7}{3}$, or $< \frac{1}{5}$. **69.** $x > \frac{1}{4}$, or $< \frac{1}{5}$.

Ex. 35. Page 133.

1. $192.$

2. $1844.$

3. $b^{m-1}.$

4. $d^{a+1}.$

5. $a^{5x-2y}.$

6. $p^{a+6b}.$

7. $(a+b)^{m+n}.$

8. $(x-y)^{m}.$

9. $m^{2x}-2m^{x}n^{y}+n^{2y}.$

10. $p^{2(m-1)}+2p^{m-1}q^{n-2}+q^{2(n-2)}.$

11. $a^{3m}+3a^{2m}b^{n}+3a^{m}b^{2n}+b^{3n}.$

12. $m^{2a}+2m^{a}n^{b}+2m^{a}p^{c}+n^{2b}+2n^{b}p^{c}+p^{2c}.$

13. $y^{2n+4}.$

14. $d^{2n+4}.$

15. $c^{2(m+n)}.$

16. $-a^{2m+7}.$

17. $-m^{2p+7}.$

18. $x^{2m+6}.$

19. $x^{2n+2}y^{2n+2}.$

20. $-\frac{x^{15}}{y^{15}}.$

21. $-\frac{a^{10}}{b^{10}}.$

22. $-\frac{n}{m}.$

23. $-\frac{b^{12}}{x^{3}c^{9}}.$

24. $-\frac{x}{y}.$

25. $\frac{n^{2}}{m^{2}}.$

26. $-\frac{a^{19}x^{3}}{b^{3}y^{19}}.$

27. $\left(\frac{m-5}{7-c}\right)^{2}.$

28. $\left(\frac{x-y}{m-p}\right)^{2}.$

29. $\frac{a-8}{x-y}.$

30. $-\frac{a-b}{x-y}.$

31. $\frac{a-b}{x-y}.$

32. $-\left(\frac{x^{4}-y^{4}}{a^{2}-b^{2}}\right)^{2}.$

33. $\frac{x^{6}-y^{6}}{a^{2}-b^{2}}(x^{3}+y^{3})^{3}.$

34. $(x^{5}+y^{5})^{m}\times(m^{8}-n^{8})^{m}.$

35. $a^{12}.$

36. $b^{30}.$

37. $x^{2m+2}.$

38. $a^{6}.$

39. $-a^{6}.$

40. $-m^{21}.$

41. $y^{6m}.$

42. $-y^{6m}.$

43. $y^{6m}.$

44. $-b^{4m-2}.$

45. $-b^{2n(2n-1)}.$

46. $m^{5n}p^{5}.$

47. $25a^{2n-2}b^{4}.$

48. $27a^{9}b^{3n-6}.$

49. $343a^{6}x^{3n-6}y^{3m+3}.$

50. $512a^{3n}b^{3n-3}c^{3n-6}.$

51. $\frac{a^{4}b^{6}}{c^{8}d^{10}}.$

52. $a^{3}x^{7}.$

53. $\frac{1}{b^{3}}.$

54. $\frac{1}{a^{8}b^{2}}.$

55. $abcde.$

56. $\left(\frac{8x-6y}{4x-3y}\right)^{10a^{2}}=1024^{a^{2}}.$

57. $\frac{m^{16}q^{39}}{n^{12}p^{2}}.$

58. $\frac{a^{20}c^{3}x^{6}}{b^{9}}.$

Ex. 36. Page 134.

1. c^{x-1}.
2. m^x.
3. m^{y-1}.
4. m^x.
5. x^{a-3b}.
6. x^{2b}.
7. $x^{2(a-b)}$.
8. a^{x+5y}.
9. $(x+y)^{2(a-b)}$.
10. $a^{p-q}b^{p+q}$.
11. $x^{3p}+x^{2p}+x^p+1$.
12. $2-x^p+3x^{2p}$.
13. $a^{n-1}+a^{n-2}b+a^{n-3}b^2+\cdots\cdots+ab^{n-2}+b^{n-1}$.
14. $a^{2n-1}-a^{2n-2}b+a^{2n-3}b^2-\cdots\cdots+ab^{2n-2}-b^{2n-1}$.
15. a^n.
16. $(\frac{2}{3}ac^2)^n$.
17. $(5a^2-3b^2)^x$.
18. $(8m+7n)^b$.
19. 64.
20. 64.
21. 243.
22. $\frac{2mn^6}{15pq^6}$.
23. $\frac{p-q}{27(m+n)^3}$.
24. 1.
25. $\frac{1}{a^3}$.
26. 1.
27. a^4.
28. $\frac{1}{x^0}=1$.
29. 1.
30. a^8.
31. a^8.
32. $a^5\times q^0=a^5$.
33. $\frac{1}{m^3}$.
34. $\frac{1}{m^3}$.
35. $\frac{1}{m^3}$.
36. b^m.
37. b^m.
38. $\frac{b^m}{d^5}$.
39. 1.
40. 1.
41. 1.
42. 2.
43. 2.
44. 1.
45. 0.
46. 2.
47. 1.
48. ∞.
49. ∞.
50. $\frac{1}{a^2}$.
51. $\frac{1}{x^4}$.
52. $-\frac{1}{x^3}$.
53. $-\frac{1}{x^{15}}$.
54. a^5.
55. m^2.
56. -1.
57. 1.
58. 1.
59. $\frac{1}{b^n}$.
60. b^n.
61. $\frac{1}{x^n}$.
62. $\frac{1}{x^n}$.
63. $\frac{n^3}{m^5}$.
64. $\left(\frac{b}{a}\right)^x$.
65. $\frac{b^n}{a^m}$.
66. $\frac{1}{m}$.
67. a^{n+1}.
68. p^{1-m}.
69. $\frac{a^m b^3}{a^2 b^n}$.
70. n^4.
71. $\frac{y}{x}$.
72. 1.
73. 1.
74. $\frac{1}{a}$.
75. q^2.
76. $\frac{m}{1-m}$.
77. n.
78. $\frac{1}{1-x}$.
79. $\left(\frac{1}{1-y}\right)^2$.
80. $(m-n)^4$.
81. $\frac{1}{(1-a)^3}$.
82. $\frac{3}{5}abc$.

83. $\frac{y^4}{5x^2}$. **84.** $\frac{1}{b^2c^3}$. **85.** $\left(\frac{y}{x}\right)^2$. **86.** a^3. **87.** $\frac{m^xn^3}{mn^x}$.

88. $\frac{a^6}{q^{4x}}$.

89. $\left(\frac{m^5n^3}{m^{3x}n^3-m^5n^{5x}}\right)^x$.

90. $\left(\frac{b^{2y}-a^{3x}}{a^{3x}b^{2y}}\right)^3$.

91. $\left(\frac{x^my^n}{x^3y^n+x^my^4}\right)^2$.

92. $\frac{a^4b^4}{(b+a)^3(b-a)}$.

93. $\frac{(n+m)^2(n^2-m^2)}{m^4n^4}$.

94. $3m^{-1}$. **96.** $-xz^{-3}$. **98.** $-bp^{-4}$.

95. $x(1-y)^{-3}$. **97.** bp^{-4}. **99.** a^2.

100. $a^{3m}(xy^2)^{-3}$. **104.** $(x^{-2}+y^{-1})^{-2}\times(x^{-3}-y^{-4})^{-3}$.

101. $b^nb^{-1}y^{-5}=b^{n-1}y^{-5}$. **105.** $(x+y)^{-2}(x^2+y^2)$.

102. $c^4c^{-m}z^{-p}=c^{4-m}z^{-p}$. **106.** $3^{-1}(mn)^2$.

103. $x^{m-5}+y^{n-3}$. **107.** 1. **108.** 2.

109. 18. **116.** $\frac{4}{5}$. **123.** 400. **130.** $\frac{1}{4}$.

110. $-\frac{5}{64}$. **117.** 1. **124.** 80. **131.** 0.

111. -375. **118.** 1. **125.** 405. **132.** 3119.

112. 1. **119.** 1. **126.** -1. **133.** 525.

113. $\frac{1}{2}$. **120.** 1. **127.** 1. **134.** -17.

114. 1. **121.** 4000. **128.** 5. **135.** 1.

115. $\frac{1}{25}$. **122.** 100,000. **129.** 16. **136.** 28.

Ex. 37. Page 138.

1. a. **7.** b^{-1}. **13.** $x^{-(m+p)}$. **19.** a^{-2}.

2. m^4. **8.** m^2n^{-2}. **14.** $y^{-(a+c)}$. **20.** $-b^{-3}$.

3. x^4. **9.** a^3b^{n-m}. **15.** x^{-5m}. **21.** $-x^{-7}$.

4. z^{-2}. **10.** $a^{-(m+n)}$. **16.** y^{-6x}. **22.** y^{-10}.

5. a^{-1}. **11.** a^{-7}. **17.** $a^{-(2b+1)}$. **23.** $-m^{-5}$.

6. h^{m-n}. **12.** b^{-7}. **18.** $a^{-2(x+y)}$. **24.** x^{-4}.

25. x^{-9}.

26. $-x^{-9}$.

27. y^{-2m}.

28. $a^{-7}+a^{-6}+a^{-5}+a^{-4}$.

29. $-y^{-10}-y^{-9}-y^{-8}-y^{-7}-y^{-6}-y^{-5}$.

30. $\left(\dfrac{a}{b}\right)^{-3}$.

31. $\dfrac{y^{-2}}{x^{-3}}$.

32. $\dfrac{m^{-1}n^{-2}}{b^{-3}}$.

33. $x^{-1}-x^{-2}+5x^{-4}-3x^{-5}$.

34. $12^{-2}=\frac{1}{144}$.

35. $30^{-1}=\frac{1}{30}$.

36. $40^{-2}=\frac{1}{1600}$.

37. $6^{-3}=\frac{1}{216}$.

38. $10^{-4}=\frac{1}{10000}$.

39. $\frac{1}{9}$.

40. $\frac{1}{9}$.

41. $\frac{1}{27}$.

42. $\frac{1}{27}$.

43. 4.

44. 10,000.

45. $(ac)^{-1}$.

46. $(\frac{1}{3}xy)^{-4}$.

47. $(a-b)^{-8}$.

48. $(x^2-y^2)^{-16}$.

49. $(3m^5-2n)^{-4}$.

50. $-(1-x^2)^{-14}$.

51. $\left(\dfrac{a^4}{b^2}-\dfrac{m^{-2}}{n^{-4}}\right)^{-3}$.

52. $\left(\dfrac{x^2}{y^6}-\dfrac{z^{-2}}{y^{-3}}\right)^{-7}$.

53. $\left(\dfrac{9x^2}{a^2}-\dfrac{4m^{-2}}{b^{-4}}\right)^{-2}$.

54. $\left(\dfrac{a^{-2}}{z^4}-\dfrac{x^{-6}}{y^{-8}}\right)^{-3}$.

55. $\left(\dfrac{4}{y^{-6}}-\dfrac{1}{z^{-2}}\right)^{-6}$.

Ex. 38. Page 140.

1. $2^{-6}=\frac{1}{64}$.

2. $2^{-6}=\frac{1}{64}$.

3. $5^{12}=244,140,625$.

4. $\frac{1}{1000000000}$.

5. 729.

6. 125.

7. 4096.

8. 1,000,000.

9. $3\frac{3}{8}$.

10. 625.

11. $\frac{2}{9}$.

12. $3\frac{3}{4}$.

13. $64a^6$.

14. $\dfrac{a^{-4}}{256}$.

15. x^6.

16. y^{-4}.

17. 1.

18. a^{-2}.

19. b^{-4}.

20. $-m^{-3}$.

21. $a^{-4}b^2$.

22. mn^3.

23. $a^{-4}b^{-10}$.

24. $-a^3b^6$.

25. $-x^{10}y^{20}$.

26. x^2y^4.

27. a^{mn}.

28. $b^{-3}n^{-3}$.

29. d^{5p}.

30. c^{-2x}.

31. m^{-2xy}.

32. a^{-2y}.

33. $a^{2m}b^{-2x}$.

34. b^3c^{3y}.

35. $x^{-2an}y^{-2mn}$.

36. $3^{-2x}a^{4x}$.

37. $-a^{-6}$.

38. $a^{-4n(2n+1)}$.

39. a^{-6n}.

40. m^{-2pqx}.

41. $\frac{a^2b^{-4}}{c^{2m}d^{2n}}$.

42. $\frac{b^{-m}c^{3m}}{x^{2m}y^{-4m}}$.

43. $\frac{xy^{-1}}{a^{-1}}$.

44. $\frac{m^{-1}}{x^py^q}$.

45. $\frac{m-n}{m+n}$.

46. $\left(\frac{p+q}{m+n}\right)^2$.

47. $\left(\frac{a+b}{a-b}\right)^2$.

48. $\left(\frac{x-y}{x+y}\right)$.

49. $\frac{m+n}{m-n}$.

50. $1-x^2$.

51. 1.

52. $\left(\frac{x+y}{a-b}\right)^5$.

53. $\left(\frac{c-d}{a+b}\right)^2$.

54. $\left(\frac{y-x}{a+b}\right)$.

55. $\frac{c-d}{m-n}$.

56. 2.

57. 27.

58. 25.

59. $\frac{1}{49}$.

60. $\frac{1}{1296}$.

61. $\frac{1}{100}$.

62. $\frac{1}{2401}$.

63. 81.

64. $\frac{1}{81}$.

65. $\frac{1}{16807}$.

66. 1.

67. $\frac{16}{9}$.

68. $\frac{8}{27}$.

69. $\frac{1}{32}$.

70. $15\frac{5}{8}$.

71. 0.09.

72. 0.49.

73. $15\frac{5}{8}$.

74. 1000.

75. 125.

76. a^3.

77. $\frac{1}{a}$.

78. $\frac{1}{a^3}$.

79. m^x.

80. $\frac{m^x}{m^3}$.

81. x^a.

82. $\frac{y^m}{y^a}$.

83. b^2.

84. $\frac{1}{c^{6x}}$.

85. $\frac{1}{(a+b)^{2y}}$.

86. $\frac{ab^2n}{m^4}$.

87. $\frac{1}{a^3}+\frac{1}{b^2}$.

88. $-\frac{1}{x^m}(x^{m-1}+x^{m-2}+\cdots\cdots+x+1)$.

89. $-x^m-x^{m-1}-x^{m-2}-\cdots\cdots$

90. 256.

91. $\frac{32}{243}$.

92. $\frac{1}{8}$.

93. 32.

94. $\frac{1}{81}$.

95. 128.

96. $\frac{1}{81}$.
97. $15\frac{5}{8}$.
98. $\frac{49}{36}$.
99. $\frac{1}{1000000}$.
100. $\frac{8}{27}$.
101. $\frac{81}{16}$.
102. $\frac{9}{16}$.
103. 1.
104. $\frac{243}{1024}$.
105. 1.
106. $\frac{1}{a^{10}b^{14}}$.
107. $\frac{1}{x^3}-\frac{1}{y}$.

Ex. 39. Page 143.

1. $a^{\frac{3}{2}}$.
2. $a^{\frac{5}{2}}$.
3. a^3.
4. $n^{\frac{5}{3}}$.
5. $a^{\frac{5}{4}}$.
6. $a^{\frac{3}{2}}b^{\frac{1}{2}}$.
7. $ab^{\frac{2}{3}}$.
8. $a^{\frac{1}{3}}b^{\frac{5}{3}}$.
9. $a-b$.
10. $m-p$.
11. $\frac{3b^{\frac{4}{3}}}{a^{\frac{2}{3}}}$.
12. $\frac{m^{\frac{5}{3}}}{n^{\frac{4}{3}}}$.
13. $\frac{a^{\frac{3}{2}}}{b^2}$.
14. $x^{\frac{5}{21}}$.
15. $m^{\frac{1}{3}}$.
16. $a^{\frac{5}{6}}$.
17. $a^{\frac{7}{12}}$.
18. $\sqrt[3]{a}$.
19. $\sqrt[5]{1}=1$.
20. $\sqrt[4]{m^3}$.
21. $\sqrt[7]{x^3}$.
22. $\sqrt[3]{n^2}$.
23. $\sqrt[4]{b^3}$.
24. $\sqrt[6]{e^5}$.
25. $\sqrt[9]{h}$.
26. $\sqrt[b]{x^a}$.
27. $\sqrt[m]{y}$.
28. $\sqrt[3]{\frac{1}{a}}$.
29. $\sqrt[4]{\frac{1}{c^3}}$.
30. $\sqrt[n]{\frac{1}{e^m}}$.
31. $\sqrt[3]{\frac{1}{x^{2m}}}$.
32. $\sqrt[m+1]{\frac{1}{y^{a+b}}}$.
33. $\frac{z^p}{\sqrt[3]{z}}=\sqrt[3]{z^{3p-1}}$.
34. $\sqrt[3]{(x^3+y^3)^2}$.
35. $\sqrt{x^5y^3}$.
36. $\frac{1}{\sqrt[3]{m^3-n^3}}$.
37. $\frac{\sqrt{a}}{b}$.
38. $\frac{m^2}{\sqrt[5]{n^8}}$.
39. $\sqrt[7]{\frac{x^{15}}{y^{20}}}$.
40. $\sqrt{\frac{1}{a^6b^6}}=\frac{1}{a^3b^3}$.
41. 2.
42. 3.
43. 5.
44. 2.
45. 3.
46. 2.
47. 5.
48. 6.
49. 7.
50. 9.
51. 32.
52. 729.
53. 3125.
54. 16.
55. 243.
56. 64.
57. 64.
58. 16.
59. $\frac{1}{8}$.
60. $\frac{1}{16}$.
61. $\frac{1}{64}$.
62. $\frac{1}{32}$.
63. $\frac{1}{8}$.
64. $\frac{1}{9}$.
65. $\frac{1}{125}$.
66. $\frac{5}{6}$.
67. $\frac{2}{3}$.
68. $\frac{5}{6}$.
69. $\frac{2}{3}$.
70. $\frac{1}{2}$.
71. $\frac{6}{5}$.
72. $\frac{3}{2}$.
73. $\frac{6}{5}$.
74. $\frac{3}{2}$.
75. 2.
76. $\frac{4}{9}$.
77. $\frac{5}{2}$.
78. $\frac{27}{8}$.

79. $\frac{8}{27}$.
80. 0.5.
81. 0.09.
82. $\frac{1}{0.5}=2$.
83. +0.09.
84. −0.008.

Ex. 40. Page 145.

1. $4a^{\frac{3}{2}}$.
2. $\frac{b^{\frac{3}{4}}}{2}+4b^{-\frac{3}{4}}=\frac{b^{\frac{3}{2}}+8}{2b^{\frac{3}{4}}}$.
3. $5(m^{\frac{1}{3}}-2m^{-\frac{1}{3}})$.
4. $9x^{-3}$.
5. $0.81m^{-\frac{1}{2}}=\left(\frac{0.9}{m}\right)^2$.
6. $0.4c^{-\frac{3}{2}}$.
7. $-1.3d^{-4}$.
8. $x^{-4}y^{\frac{1}{3}}-2x^{-3}y^{\frac{1}{2}}$.
9. $-(m+1)x^{-\frac{1}{3}}+(a+d)x^{-\frac{2}{3}}+(3b-c)x^{-\frac{4}{3}}$.
10. $(2-m)y^{-\frac{1}{3}}+(5b+m-c)y^{-\frac{2}{3}}+(3-c)y^{\frac{2}{3}}$.
11. $-(a+1)b^{-\frac{1}{3}}$.
12. $-3.7d^{-\frac{3}{4}}$.
13. $0.5e^{-x}$.
14. $-1.15h^{-4}$.
15. $4m^{-5}$.
16. $1.3p^{-3}$.
17. $2.4x^{-1}-7.7x^{-2}+0.1x^{-3}$.
18. $a^{\frac{1}{3}}$.
19. $b^{\frac{1}{15}}$.
20. $a^{-\frac{11}{8}}$.
21. $c^{-\frac{1}{3}}$.
22. $d^{-\frac{2}{3}}$.
23. m^{-1}.
24. $m^{-\frac{2}{7}}$.
25. $x^{\frac{m-n}{mn}}$.
26. $x^{\frac{mq+np}{nq}}$.
27. $y^{-\frac{(an+cm)}{cn}}$.
28. $a^{-\frac{x}{y}}$.
29. $z^{-\frac{c}{m}}$.
30. $u^{-\frac{1}{4}}$.
31. $d^{\frac{-(n+4)}{2m}}$.
32. $e^{\frac{-x(2y+1)}{y}}$.
33. $a^{\frac{2}{3}}$.
34. $b^{\frac{3}{2}}$.
35. $h^{\frac{1}{3}}$.
36. $m^{\frac{a}{bx}}$.
37. $n^{-\frac{ac}{bd}}$.
38. $x^r+2x^{\frac{r}{2}}y^{2p}+9y^{4p}$.
39. $x^p+x^{\frac{p}{2}}y^{\frac{q}{2}}+y^q$.
40. $x^{-2}+x^{-1}y^{-1}+y^{-2}$.
41. $c^{-\frac{7}{m}}-5c^{-\frac{6}{m}}+10c^{-\frac{5}{m}}-8c^{-\frac{4}{m}}-3c^{-\frac{3}{m}}+c^{-\frac{2}{m}}$.
42. $d^{-\frac{2}{x}}-\frac{5}{6}d^{-\frac{3}{x}}+\frac{1}{2}d^{-\frac{4}{x}}+2\frac{29}{60}d^{-\frac{5}{x}}-1\frac{3}{5}d^{-\frac{6}{x}}+\frac{4}{5}d^{-\frac{7}{x}}$.
43. $\frac{25x^{-\frac{5}{3}}}{16y^{-\frac{4}{3}}}-\frac{9a^{-1}}{4b^{-\frac{4}{3}}}$.
44. $4a^{-2}b^{-\frac{4}{3}}-12a^{-\frac{3}{2}}b^{-\frac{11}{12}}+9a^{-\frac{3}{2}}b^{-\frac{2}{3}}$.
45. $e^{-\frac{1}{4}}$.
46. $h^{-\frac{1}{3}}$.
47. $k^{-\frac{1}{3}}$.

48. $l^{\frac{1}{6}}$.

49. $g^{\frac{1}{12}}$.

50. $a^{\frac{3}{10}}$.

51. $b^{-\frac{11}{12}}$.

52. $m^{-\frac{1}{20}}$.

53. $p^{-\frac{(m+n)}{mn}}$.

54. $q^{\frac{a+b}{ab}}$.

55. $t^{-\frac{(am-n)}{an}}$.

56. $v^{-\frac{(b+ap)}{bp}}$.

57. $x^{\frac{bn-2m^2}{2mn}}$.

58. $z^{-\frac{(aq+p)}{q}}$.

59. $y^{\frac{m-n}{n}}$.

60. $c^{\frac{a-2}{m}}$.

61. $d^{\frac{b-2a}{2m}}$.

62. $e^{\frac{am+n}{an}}$.

63. $a^{\frac{3}{2}}b^{-\frac{3p}{q}}$.

64. $\frac{a^{-\frac{1}{4}}}{3b^{-\frac{2}{3}}}$.

65. $x+y$.

66. $x-x^{\frac{1}{2}}$.

67. $x^{\frac{1}{2}}-\frac{2}{3}x^{\frac{1}{3}}+\frac{1}{2}x^{\frac{1}{4}}$.

68. $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}-x^{\frac{1}{3}}y^{\frac{1}{3}}-x^{\frac{1}{3}}z^{\frac{1}{3}}-y^{\frac{1}{3}}z^{\frac{1}{3}}$.

Ex. 41. Page 147.

1. $a^3+3a^2b+3ab^2+b^3$.

2. $m^5+5m^4n+10m^3n^2+10m^2n^3+5mn^4+n^5$.

3. $c^8+8c^7d+28c^6d^2+56c^5d^3+70c^4d^4+56c^3d^5+28c^2d^6$
$+8cd^7+d^8$.

4. $h^{10}+10h^9l+45h^8l^2+120h^7l^3+210h^6l^4+252h^5l^5$
$+210h^4l^6+120h^3l^7+45h^2l^8+10hl^9+l^{10}$.

5. $x^{11}+11x^{10}y+55x^9y^2+165x^8y^3+330x^7y^4+462x^6y^5$
$+462x^5y^6+330x^4y^7+165x^3y^8+55x^2y^9+11xy^{10}+y^{11}$.

6. $y^7-7y^6z+21y^5z^2-35y^4z^3+35y^3z^4-21y^2z^5+7yz^6-z^7$.

7. $x^9-9x^8a+36x^7a^2-84x^6a^3+126x^5a^4-126x^4a^5$
$+84x^3a^6-36x^2a^7+9xa^8-a^9$

8. $y^3-3y^2b+3yb^2-b^3$.

9. $c^5-5c^4d+10c^3d^2-10c^2d^3+5cd^4-d^5$.

10. $m^8-8m^7a+28m^6a^2-56m^5a^3+70m^4a^4-56m^3a^5$
$+28m^2a^6-8ma^7+a^8$.

11. $1+4a+6a^2+4a^3+a^4$.

12. $a^4 - 4a^3 + 6a^2 - 4a + 1.$

13. $1 + 6d + 15d^2 + 20d^3 + 15d^4 + 6d^5 + d^6.$

14. $1 - 8h + 28h^2 - 56h^3 + 70h^4 - 56h^5 + 28h^6 - 8h^7 + h^8.$

15. $1 + 10x + 45x^2 + 120x^3 + 210x^4 + 252x^5 + 210x^6$
$+ 120x^7 + 45x^8 + 10x^9 + x^{10}.$

16. $32 + 80a + 80a^2 + 40a^3 + 10a^4 + a^5.$

17. $729 - 1458a + 1215a^2 - 540a^3 + 135a^4 - 18a^5 + a^6.$

18. $m^9 - 18m^8 + 144m^7 - 672m^6 + 2016m^5 - 4032m^4$
$+ 5376m^3 - 4608m^2 + 2304m - 512.$

19. $x^3 - 15x^2 + 75x - 125.$

20. $y^5 - 15y^4 + 90y^3 - 270y^2 + 405y - 243.$

21. $a^{21} + 7a^{18}b^2 + 21a^{15}b^4 + 35a^{12}b^6 + 35a^9b^8 + 21a^6b^{10}$
$+ 7a^3b^{12} + b^{14}.$

22. $m^{40} - 8m^{35}n^2 + 28m^{30}n^4 - 56m^{25}n^6 + 70m^{20}n^8 - 56m^{15}n^{10}$
$+ 28m^{10}n^{12} - 8m^5n^{14} + n^{16}.$

23. $x^{15} + 5x^{12}y^4 + 10x^9y^8 + 10x^6y^{12} + 5x^3y^{16} + y^{20}.$

24. $a^{32} - 8a^{28}y^3 + 28a^{24}y^6 - 56a^{20}y^9 + 70a^{16}y^{12} - 56a^{12}y^{15}$
$+ 28a^8y^{18} - 8a^4y^{21} + y^{24}.$

25. $a^7 - 7a^6b^2 + 21a^5b^4 - 35a^4b^6 + 35a^3b^8 - 21a^2b^{10}$
$+ 7ab^{12} - b^{14}.$

26. $\frac{a^2}{b^2} + \frac{2ax}{by} + \frac{x^2}{y^2}.$ 27. $\frac{a^3}{c^3} - \frac{3a^2d}{c^2x} + \frac{3ad^2}{cx^2} - \frac{d^3}{x^3}.$

28. $\frac{m^5}{n^5} + \frac{5m^4p}{n^4q} + \frac{10m^3p^2}{n^3q^2} + \frac{10m^2p^3}{n^2q^3} + \frac{5mp^4}{nq^4} + \frac{p^5}{q^5}.$

29. $\frac{m^6}{p^6} - \frac{6m^5n}{p^5q} + \frac{15m^4n^2}{p^4q^2} - \frac{20m^3n^3}{p^3q^3} + \frac{15m^2n^4}{p^2q^4} - \frac{6mn^5}{pq^5} + \frac{n^6}{q^6}.$

30. $\frac{1}{a^4} - \frac{4b}{a^3c} + \frac{6b^2}{a^2c^2} - \frac{4b^3}{ac^3} + \frac{b^4}{c^4}.$

31. $m^{\frac{10}{3}} - 5m^{\frac{8}{3}}n^{\frac{1}{2}} + 10m^2n - 10m^{\frac{4}{3}}n^{\frac{3}{2}} + 5m^{\frac{2}{3}}n^2 - n^{\frac{5}{2}}$.

32. $a^{\frac{9}{2}} + 6a^{\frac{15}{4}}b^{\frac{2}{3}} + 15a^3b^{\frac{4}{3}} + 20a^{\frac{9}{4}}b^2 + 15a^{\frac{3}{2}}b^{\frac{8}{3}} + 6a^{\frac{3}{4}}b^{\frac{10}{3}} + b^4$.

33. $x^{\frac{14}{5}} + 7x^{\frac{12}{5}}y^{\frac{1}{4}} + 21x^2y^{\frac{1}{2}} + 35x^{\frac{8}{5}}y^{\frac{3}{4}} + 35x^{\frac{6}{5}}y + 21x^{\frac{4}{5}}y^{\frac{5}{4}}$
$+ 7x^{\frac{2}{5}}y^{\frac{3}{2}} + y^{\frac{7}{4}}$.

34. $x^2 - 4x^{\frac{3}{2}}z^{\frac{1}{3}} + 6xz^{\frac{2}{3}} - 4x^{\frac{1}{2}}z + z^{\frac{4}{3}}$.

35. $a + 5a^{\frac{4}{5}}b^{\frac{2}{5}} + 10a^{\frac{3}{5}}b^{\frac{4}{5}} + 10a^{\frac{2}{5}}b^{\frac{6}{5}} + 5a^{\frac{1}{5}}b^{\frac{8}{5}} + b^2$.

36. $a^{-5} + 5a^{-4}b^{-2} + 10a^{-3}b^{-4} + 10a^{-2}b^{-6} + 5a^{-1}b^{-8} + b^{-10}$.

37. $a^{-24} + 8a^{-21}x^{-1} + 28a^{-18}x^{-2} + 56a^{-15}x^{-3} + 70a^{-12}x^{-4}$
$+ 56a^{-9}x^{-5} + 28a^{-6}x^{-6} + 8a^{-3}x^{-7} + x^{-8}$.

38. $m^{-28} - 7m^{-24}n^{-1} + 21m^{-20}n^{-2} - 35m^{-16}n^{-3} + 35m^{-12}n^{-4}$
$- 21m^{-8}n^{-5} + 7m^{-4}n^{-6} - n^{-7}$.

39. $d^{-12} - 6d^{-10}x^2 + 15d^{-8}x^4 - 20d^{-6}x^6 + 15d^{-4}x^8$
$- 6d^{-2}x^{10} + x^{12}$.

40. $c^{-20} + 4c^{-15}z + 6c^{-10}z^2 + 4c^{-5}z^3 + z^4$.

41. $m^{-\frac{5}{2}} + 5m^{-2}n^{-\frac{2}{3}} + 10m^{-\frac{3}{2}}n^{-\frac{4}{3}} + 10m^{-1}n^{-2} + 5m^{-\frac{1}{2}}n^{-\frac{8}{3}}$
$+ n^{-\frac{10}{3}}$.

42. $n^{-\frac{15}{4}} + 5n^{-3}p^{-\frac{2}{3}} + 10n^{-\frac{9}{4}}p^{-\frac{4}{3}} + 10n^{-\frac{3}{2}}p^{-2} + 5n^{-\frac{3}{4}}p^{-\frac{8}{3}} + p^{-\frac{10}{3}}$.

43. $n^{-\frac{8}{5}} - 4n^{-\frac{6}{5}}p^{-\frac{3}{5}} + 6n^{-\frac{4}{5}}p^{-\frac{6}{5}} - 4n^{-\frac{2}{5}}p^{-\frac{9}{5}} + p^{-\frac{12}{5}}$.

44. $a^{-\frac{5}{2}} + 5a^{-2}b^{-\frac{1}{3}} + 10a^{-\frac{3}{2}}b^{-\frac{2}{3}} + 10a^{-1}b^{-1} + 5a^{-\frac{1}{2}}b^{-\frac{4}{3}} + b^{-\frac{5}{3}}$.

45. $x^{-\frac{3}{4}} + 3x^{-\frac{1}{2}}y + 3x^{-\frac{1}{4}}y^2 + y^3$.

46. $y^{-12} - 6y^{-10}z^{-\frac{2}{3}} + 15y^{-8}z^{-\frac{4}{3}} - 20y^{-6}z^{-2} + 15y^{-4}z^{-\frac{8}{3}}$
$- 6y^{-2}z^{-\frac{10}{3}} + z^{-4}$.

47. $u^{-\frac{8}{5}} + 4u^{-\frac{6}{5}}v^3 + 6u^{-\frac{4}{5}}v^6 + 4u^{-\frac{2}{5}}v^9 + v^{12}$.

48. $1 - 5z^{-\frac{2}{3}} + 10z^{-\frac{4}{3}} - 10z^{-2} + 5z^{-\frac{8}{3}} - z^{-\frac{10}{3}}$.

49. $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$.

50. $243y^5 - 405y^4z + 270y^3z^2 - 90y^2z^3 + 15yz^4 - z^5$.

51. $4096z^6 - 6144z^5u + 3840z^4u^2 - 1280z^3u^3 + 240z^2u^4$
$- 24zu^5 + u^6.$

52. $128a^7 - 448a^6b^2 + 672a^5b^4 - 560a^4b^6 + 280a^3b^8$
$- 84a^2b^{10} + 14ab^{12} - b^{14}.$

53. $m^{12} + 18m^{10}b + 135m^8b^2 + 540m^6b^3 + 1215m^4b^4$
$+ 1458m^2b^5 + 729b^6.$

54. $a^5 - 10a^4c + 40a^3c^2 - 80a^2c^3 + 80ac^4 - 32c^5.$

55. $x^{12} - 12x^9d + 54x^6d^2 - 108x^3d^3 + 81d^4.$

56. $u^{-2} - 12u^{-\frac{4}{3}}e + 48u^{-\frac{2}{3}}e^2 - 64e^3.$

57. $128a^{14} - 1344a^{12}b + 6048a^{10}b^2 - 15{,}120a^8b^3$
$+ 22{,}680a^6b^4 - 20{,}412a^4b^5 + 10{,}206a^2b^6 - 2187b^7.$

58. $729x^{-6} + 2916x^{-5}y^{\frac{2}{3}} + 4860x^{-4}y^{\frac{4}{3}} + 4320x^{-3}y^2$
$+ 2160x^{-2}y^{\frac{8}{3}} + 576x^{-1}y^{\frac{10}{3}} + 64y^4.$

59. $256x^8 + 3072x^7z^{-\frac{1}{2}} + 16{,}128x^6z^{-1} + 48{,}384x^5z^{-\frac{3}{2}}$
$+ 90{,}720x^4z^{-2} + 108{,}864x^3z^{-\frac{5}{2}} + 81{,}648x^2z^{-3}$
$+ 34{,}992xz^{-\frac{7}{2}} + 6561z^{-4}.$

60. $\frac{1}{32}(a^5 - 15a^4d^{-1} + 90a^3d^{-2} - 270a^2d^{-3} + 405ad^{-4} - 243d^{-5}).$

61. $3125m^{-10} + 625m^{-8}n^{-1} + 50m^{-6}n^{-2} + 2m^{-4}n^{-3}$
$+ \frac{m^{-2}n^{-4}}{25} + \frac{n^{-5}}{3125}.$

62. $\frac{64a^6}{b^6} - \frac{576a^5c}{b^5d} + \frac{2160a^4c^2}{b^4d^2} - \frac{4320a^3c^3}{b^3d^3} + \frac{4860a^2c^4}{b^2d^4}$
$- \frac{2916ac^5}{bd^5} + \frac{729c^6}{d^6}.$

63. $\frac{1}{32a^5} + \frac{15}{16a^4b} + \frac{45}{4a^3b^2} + \frac{135}{2a^2b^3} + \frac{405}{2ab^4} + \frac{243}{b^5}.$

64. $\frac{16}{a^{-4}} + \frac{32d^{-2}}{a^{-3}c^{-3}} + \frac{24d^{-4}}{a^{-2}c^{-6}} + \frac{8d^{-6}}{a^{-1}c^{-9}} + \frac{d^{-8}}{c^{-12}}.$

65. $\frac{x^7}{128} - \frac{7x^6y^{\frac{1}{3}}}{192} + \frac{7x^5y^{\frac{2}{3}}}{96} - \frac{35x^4y}{432} + \frac{35x^3y^{\frac{4}{3}}}{648} - \frac{7x^2y^{\frac{5}{3}}}{324}$
$+ \frac{7xy^2}{1458} - \frac{y^{\frac{7}{3}}}{2187}.$

66. $\frac{a^5}{243b^5} + \frac{10a^4m^{\frac{1}{3}}}{81b^4x} + \frac{40a^3m^{\frac{2}{3}}}{27b^3x^2} + \frac{80a^2m}{9b^2x^3} + \frac{80am^{\frac{4}{3}}}{3bx^4} + \frac{32m^{\frac{5}{3}}}{x^5}.$

67. $1 - \frac{9y}{2} + \frac{135y^2}{16} - \frac{135y^3}{16} + \frac{1215y^4}{256} - \frac{729y^5}{512} + \frac{729y^6}{4096}.$

68. $\frac{2187a^7}{b^7} - \frac{10206a^8}{b^6x} + \frac{20412a^9}{b^5x^2} - \frac{22680a^{10}}{b^4x^3} + \frac{15120a^{11}}{b^3x^4}$
$- \frac{6048a^{12}}{b^2x^5} + \frac{1344a^{13}}{bx^6} - \frac{128a^{14}}{x^7}.$

69. $625m^{4x} - \frac{500m^{3x}n^{2y}}{3} + \frac{50m^{2x}n^{4y}}{3} - \frac{20m^xn^{6y}}{27} + \frac{n^{8y}}{81}.$

70. $243b^{5(m+1)} + 810b^{4(m+1)}d^{n-1} + 1080b^{3(m+1)}d^{2(n-1)}$
$+ 720b^{2(m+1)}d^{3(n-1)} + 240b^{m+1}d^{4(n-1)} + 32d^{5(n-1)}.$

71. $256x^{4(a-3)} - 768x^{3(a-3)}y^{2-b} + 864x^{2(a-3)}y^{2(2-b)}$
$- 432x^{a-3}y^{3(2-b)} + 81y^{4(2-b)}.$

72. $125m^{3(2x-1)} + \frac{225}{4}m^{2(2x-1)}n^{y-2} + \frac{135}{16}m^{2x-1}n^{2(y-2)} + \frac{27}{64}n^{3(y-2)}.$

73. $4a^6b^2 - \frac{20}{3}a^3b^4m + \frac{25}{9}m^2b^6.$

74. $\frac{a^{4m}b^{4x}}{256} - \frac{a^{4m+1}b^{4x-1}}{48} + \frac{a^{4m+2}b^{4x-2}}{24} - \frac{a^{4m+3}b^{4x-3}}{27} + \frac{a^{4m+4}b^{4x-4}}{81}$

75. $32a^{10}b^{5m} + 40a^{m+8}b^{4m+2} + 20a^{2m+6}b^{3m+4} + 5a^{3m+4}b^{2m+6}$
$+ \frac{5}{8}a^{4m+2}b^{m+8} + \frac{1}{32}a^{5m}b^{10}.$

Ex. 42. Page 148.

1. $\sqrt{8}$.
2. $\sqrt{175}$.
3. $\sqrt{180}$.
4. $\sqrt{84}$.
5. $\sqrt[3]{54}$.
6. $\sqrt[3]{1029}$.
7. $\sqrt[3]{2560}$.
8. $\sqrt[4]{1024}$.
9. $\sqrt[5]{96}$.

10. $\sqrt[7]{4374}$.
11. $\sqrt[3]{4000}$.
12. $\sqrt[3]{7290}$.
13. $\sqrt{2624}$.
14. $\sqrt{5a^2}$.
15. $\sqrt[4]{2x^4}$.
16. $\sqrt[5]{6y^5}$.
17. $\sqrt[3]{7z^3}$.
18. $\sqrt[4]{6b^4}$.
19. $\sqrt[5]{-7d^5}$.
20. $-\sqrt[6]{10m^6}$.
21. $\sqrt[3]{-15n^3}$.
22. $-\sqrt{9a}$.
23. $\sqrt[3]{-125b}$.
24. $\sqrt[5]{-32x}$.
25. $-\sqrt[4]{256y}$.
26. $\sqrt{9bx}$.
27. $\sqrt{a^2b}$.
28. $\sqrt{m^4yz^3}$.
29. $\sqrt[5]{n^{15}xy^2}$.
30. $-\sqrt[4]{a^{12}m^2n}$.
31. $\sqrt[3]{-x^{15}y^4z}$.
32. $\sqrt{\frac{a}{4}}$.
33. $\sqrt[3]{\frac{8m^2}{27}}$.
34. $2\sqrt{2}$.
35. $2\sqrt{3}$.
36. $2\sqrt{7}$.
37. $5\sqrt{2}$.
38. $6\sqrt{2}$.
39. $2\sqrt[3]{9}$.
40. $5\sqrt[3]{4}$.
41. $3\sqrt[3]{4}$.
42. $4\sqrt[3]{3}$.
43. $2\sqrt[3]{7}$.
44. $2\sqrt[5]{5}$.
45. $4\sqrt[5]{2}$.
46. $3\sqrt[6]{5}$.
47. $3\sqrt[4]{7}$.
48. $2\sqrt[4]{7}$.
49. $18\sqrt{5}$.
50. $18\sqrt[3]{4}$.
51. $10\sqrt[4]{9}$.
52. $a\sqrt{b}$.
53. $m\sqrt[3]{n^2}$.
54. $b^2\sqrt[4]{a^3}$.
55. $b\sqrt[4]{c}$.
56. $c^3\sqrt[5]{x^2}$.
57. $x^2y\sqrt[3]{y}$.
58. $y^2z\sqrt{z}$.
59. $m^3n^2\sqrt[3]{n}$.
60. $xy\sqrt[4]{xy^3}$.
61. $uv^3\sqrt[3]{uv}$.
62. $yz^2\sqrt[4]{y^3}$.
63. $ab\sqrt[10]{a^2b^5}$.
64. $2a\sqrt{b}$.
65. $5m^2\sqrt{x}$.
66. $6ab\sqrt{a}$.
67. $4x^2\sqrt[3]{y}$.
68. $3m\sqrt[4]{m^2n^2}$.
69. $2ab\sqrt[6]{a^2}$.
70. $8c^2\sqrt{5d}$.
71. $\frac{2a}{3}\sqrt{ab}$.
72. $\frac{5}{6n^2}\sqrt[3]{m^2n}$.
73. $\frac{a^2}{2}\sqrt{a}$.
74. $\frac{m}{3}\sqrt[5]{mn}$.
75. $\frac{xy}{6}\sqrt[4]{xy^3}$.
76. $\frac{m-n}{7}\sqrt{y}$.
77. $\frac{x-1}{8}\sqrt[3]{z^2}$.
78. $\frac{x^2-y^2}{2}\sqrt{x-y}$.

Ex. 43. Page 150.

1. $\sqrt[6]{3^3}=\sqrt[6]{27}$; $\sqrt[6]{4}$.
2. $\sqrt[4]{6}$; $\sqrt[4]{49}$.
3. $\sqrt[6]{125}$; $\sqrt[6]{16}$.
4. $\sqrt[12]{2^4}=\sqrt[12]{16}$; $\sqrt[12]{3^3}=\sqrt[12]{27}$.
5. $\sqrt[10]{64}$; $\sqrt[10]{32}$.
6. $\sqrt[12]{256}$; $\sqrt[12]{512}$.
7. $\sqrt[12]{64}$; $\sqrt[12]{625}$; $\sqrt[12]{216}$.
8. $\sqrt[20]{16{,}807}$; $\sqrt[20]{625}$; $\sqrt[20]{14{,}400}$.
9. $\sqrt{3}$; $2\sqrt{3}$.
10. $\sqrt{5}$; $2\sqrt{5}$.
11. $3\sqrt{7}$; $\sqrt{7}$.
12. $6\sqrt{2}$; $2\sqrt{2}$.
13. $3\sqrt{11}$; $4\sqrt{11}$.
14. $3\sqrt[3]{2}$; $2\sqrt[3]{2}$.
15. $2\sqrt[3]{9}$; $3\sqrt[3]{9}$.
16. $\sqrt[4]{5}$; $3\sqrt[4]{5}$.
17. $3\sqrt{2}$; $8\sqrt{2}$; $4\sqrt{2}$.
18. $21\sqrt[3]{2}$; $6\sqrt[3]{2}$; $6\sqrt[3]{2}$.
19. $3\sqrt{3}$; $8\sqrt{3}$; $18\sqrt{3}$.
20. $4\sqrt[3]{2}$; $7\sqrt[3]{2}$; $2\sqrt[3]{2}$.
21. $3\sqrt[4]{2}$; $6\sqrt[4]{2}$; $6\sqrt[4]{2}$.
22. $\frac{2}{3}\sqrt{3}$; $\frac{10}{3}\sqrt{3}$.
23. $\frac{1}{10}\sqrt{10}$; $\frac{1}{15}\sqrt{10}$.
24. $\frac{5}{7}\sqrt{6}$; $\frac{1}{33}\sqrt{6}$.
25. $\frac{1}{20}\sqrt{5}$; $\frac{1}{5}\sqrt{5}$.
26. $2\sqrt{5}$; $\frac{3}{5}\sqrt{5}$; $20\sqrt{5}$; $\frac{21}{5}\sqrt{5}$; $\frac{1}{2}\sqrt{5}$; $\frac{9}{20}\sqrt{5}$.
27. $\frac{2}{3}\sqrt[3]{9}$; $\frac{1}{2}\sqrt[3]{9}$; $\frac{2}{7}\sqrt[3]{9}$; $\frac{1}{3}\sqrt[3]{5}\sqrt[3]{9}$; $\frac{2}{3}\sqrt[3]{2}\sqrt[3]{9}$.
28. $\frac{1}{2}\sqrt[5]{6}$; $2\sqrt[5]{6}$; $\frac{1}{4}\sqrt[5]{6}$; $\frac{1}{5}\sqrt[5]{3}\sqrt[5]{6}$; $\frac{1}{3}\sqrt[5]{6}$.
29. $\frac{1}{2}\sqrt{5}$; $\frac{1}{3}\sqrt{5}$.
30. $\frac{1}{10}\sqrt{5}$; $\frac{1}{15}\sqrt{5}$.
31. $\frac{1}{2}\sqrt{3}$; $\frac{4}{9}\sqrt{3}$.
32. $\frac{1}{5}\sqrt[3]{7}$; $\frac{1}{4}\sqrt[3]{7}$.
33. $a\sqrt[6]{ab}$; $a^2b\sqrt[6]{ab}$.
34. $3m\sqrt[3]{m}$; $2m^2\sqrt[3]{m}$.
35. $x\sqrt[3]{x(x-3)}$; $\frac{1}{x}\sqrt[3]{x(x-3)}$.
36. $\frac{a^2}{b^3}\sqrt{bc}$; $\frac{ac}{bx}\sqrt{bc}$: $\frac{ax}{by}\sqrt{bc}$.

37. $\frac{a}{b^2}\sqrt{abx}$; $\frac{x^2}{b^3}\sqrt{abx}$; $\frac{x}{ab}\sqrt{abx}$.

38. $\frac{1}{a}\sqrt[4]{a}$; $\frac{b}{a}\sqrt[4]{a}$; $\sqrt[4]{a}$; $\frac{1}{cx^2}\sqrt[4]{a}$.

39. $\frac{1}{a}\sqrt{a^2-1}$; $\frac{1}{a+1}\sqrt{a^2-1}$.

40. $\frac{1}{b}\sqrt{b(1-ab)}$; $\frac{d}{c}\sqrt{b(1-ab)}$.

41. $(a-b)\sqrt{a-b}$; $\frac{a^2+b^2}{a-b}\sqrt{a-b}$.

42. $\sqrt[3]{(x-b^2)^2}$; $\frac{y-z}{x-b^2}\sqrt[3]{(x-b^2)^2}$;

$\frac{y\sqrt[3]{5y^2}}{x-b^2}\sqrt[3]{(x-b^2)^2}$.

43. $\frac{m}{1-x}\sqrt{m}$; $\frac{2(1-x)}{cn}\sqrt{m}$.

44. $4\sqrt{2}$.
45. $7\sqrt{3}$.
46. $8\sqrt{2}$.
47. $10\sqrt{5}$.
48. $3\sqrt{6}$.
49. $4\sqrt{5}$.
50. $3\sqrt{7}$.
51. $3\sqrt{11}$.
52. $\sqrt{3}+\sqrt{19}$.
53. $\sqrt{8}+\sqrt{12}$.
54. $\sqrt{10}+\sqrt{8}+\sqrt{6}$.

Ex. 44. Page 152.

1. $5\sqrt{3}$.
2. $10\sqrt{7}$.
3. $13\sqrt{10}$.
4. $4\sqrt{2}$.
5. $2\sqrt{11}$.
6. $6\sqrt{3}$.
7. $3\sqrt[3]{3}$.
8. $7\sqrt{13}$.
9. $4\sqrt{2}$.
10. $6\sqrt[3]{4}$.
11. $3\sqrt{2}$.
12. $5\sqrt[5]{2}$.
13. $\frac{1}{2}\sqrt[3]{5}$.
14. $-4\sqrt[4]{3}$.
15. $11\sqrt{7}$.
16. $-\sqrt{2}$.
17. $12\sqrt{3}$.
18. $129\sqrt{5}$.
19. $19\sqrt{5}-23\sqrt{3}$.

20. $35\sqrt{6}$.
21. $6\sqrt{5}$.
22. $4\sqrt{13}-3\sqrt{11}$.
23. $-\frac{43}{12}\sqrt{2}-1\frac{2}{3}\sqrt{5}$.
24. $4\sqrt[3]{5}-11\frac{5}{6}\sqrt[3]{4}$.
25. $\frac{73}{18}+\frac{31}{15}\sqrt{5}$.
26. $\frac{19}{12}\sqrt{2}-\frac{1}{2}\sqrt[3]{2}$.
27. $2\sqrt{15}$.
28. $6\sqrt{6}-2\sqrt{11}$.
29. $4\frac{4}{15}\sqrt{6}-\sqrt[3]{14}$.
30. $2\sqrt{a}$.
31. $-7\frac{1}{2}\sqrt[3]{a^2}$.
32. $(a+b-3)\sqrt{a}$.
33. $5\sqrt{b}$.
34. $(c+5)\sqrt[3]{c}$.
35. $(a-b+2)\sqrt[5]{b}$.
36. $(a-b)^2\sqrt{x}$.
37. $(x+y)^2\sqrt{z}$.
38. $2a(b-1)\sqrt{c}$.
39. $(7a-2)\sqrt[4]{a}$.
40. $(3m+1)\sqrt[3]{m}$.
41. $-(7+5a)\sqrt{2a}$.
42. $10ab\sqrt{7ab}$.
43. $4mn\sqrt{5m}$.
44. $(2ab-6)^2\sqrt{2b}$.
45. $2a^2(2+5b)\sqrt[3]{b}$.

Ex. 45. Page 153.

1. 9.
2. 10.
3. 6.
4. 3.
5. 4.
6. 8.
7. 3.
8. 6.
9. 2.
10. 3.
11. 8.
12. -7.
13. 11.
14. -2.
15. 27.
16. 72.
17. 25.
18. -729.
19. -60.
20. -14.
21. $3\sqrt{2}$.
22. $3\sqrt[3]{2}$.
23. $2\sqrt[4]{3}$.
24. $3\sqrt[5]{2}$.
25. $-9\sqrt[3]{5}$.
26. 8.
27. 60.
28. $\frac{3}{4}$.
29. $-\frac{3}{2}\sqrt[3]{2}$.
30. 24.
31. 10.
32. 4.
33. $-237\frac{3}{4}$.
34. $-5\sqrt[3]{6}$.
35. $\sqrt{21}$.
36. $\sqrt{7}$.
37. 8.
38. 5.
39. 7.
40. 4.
41. 4.
42. 5.
43. $\sqrt{5}-4$.
44. $2(5-\sqrt{2})$.

45. $-(15+19\sqrt{3})$.

46. $3(12-5\sqrt{6})$.

47. $6\sqrt{2}-3\sqrt{15}+8\sqrt{3}-6\sqrt{10}$.

48. $\sqrt{14}-\sqrt{6}+\sqrt{35}-\sqrt{15}$.

49. $8-8\sqrt[3]{12}+\sqrt[3]{18}$.

50. 30.

51. $54-7\sqrt{35}-26\sqrt{21}-7\sqrt{15}$.

52. $171-36\sqrt{6}$.

53. $86+23\frac{1}{2}\sqrt{6}$.

54. $140+10\sqrt{6}$.

55. $\sqrt[3]{12}-2\sqrt[3]{3}+2\sqrt[3]{18}+3\sqrt[3]{2}+4\sqrt[3]{9}-3\sqrt[3]{4}-1$.

56. $156-24\sqrt[3]{4}$.

57. $340+51\sqrt[3]{18}$.

58. $4-\frac{85}{2}\sqrt[3]{3}+25\sqrt[3]{9}$.

59. $-\sqrt{2}$.

60. $176+66\sqrt{5}$.

61. $110+24\sqrt{10}-12\sqrt{6}-16\sqrt{15}$.

62. $33-18\sqrt{2}$.

63. 125.

64. 243.

65. 5887.

66. $75\frac{3}{8}+7\frac{1}{2}\sqrt{2}$.

67. $9x^5-12x^4+4x^3$.

68. $a^2y+2aby^2+b^2y^3$.

69. $ab^3m^4-2a^2b^2m^3+a^3bm^2$.

70. $48p^2z^5-12pqz^3\sqrt{6}+\frac{9}{2}q^2z$.

71. $\sqrt[6]{2000}$.

72. $5\sqrt{30}$.

73. $2\sqrt{10}$.

74. 6.

75. 4.

76. $\sqrt[24]{2^{14}\times 5^{11}}$.

77. 2.

78. 15.

79. $\sqrt[10]{13{,}824}$.

80. $6\sqrt[3]{9}$.

81. $\sqrt[20]{81\times 42^5}$.

82. $\sqrt[15]{2^8}$.

83. $125\sqrt[7]{7^5}$.

84. $6\sqrt[30]{6\times 7^{21}}$.

85. $\sqrt[15]{\frac{1}{144}}$.

86. $\sqrt[20]{\frac{5^4\times 7}{3^6}}$.

87. $\sqrt[20]{\frac{9}{2^{15}}}=\frac{\sqrt[10]{3}}{\sqrt[4]{8}}$.

88. $\sqrt[36]{\frac{2}{3\times 7^4}}$.

89. $\sqrt[20]{\frac{256}{5}}$.

Ex. 46. Page 156.

1. 9.
2. 3.
3. 2.
4. 5.
5. 6.
6. 2.
7. 2.
8. 3.
9. 5.
10. 4.
11. $\frac{3}{2}$.
12. $\frac{5}{6}\sqrt{2}$.
13. $\frac{7}{6}$.
14. $\frac{2}{3}$.
15. $\frac{2}{7}$.
16. $1\frac{1}{6}$.
17. $1\frac{2}{9}$.
18. $1\frac{9}{17}$.
19. $\sqrt{2}+5\sqrt{6}$.
20. $7\sqrt{3}-3\sqrt{5}$.
21. $4\sqrt{35}+8\sqrt{14}$.
22. $5\sqrt{7}-4\sqrt{6}+2\sqrt{5}$.
23. $2\sqrt{42}-3\sqrt{14}+5\sqrt{10}$.
24. $2\sqrt[3]{25}-6\sqrt[3]{4}+5\sqrt[3]{36}$.
25. $5\sqrt[3]{75}+3\sqrt[3]{18}-4\sqrt[3]{36}$.
26. $3-2\sqrt{2}$.
27. $\sqrt{3}$.
28. $\sqrt{x}$.
29. $\sqrt{2}$.
30. $\sqrt{2m}$.
31. $\frac{1}{2}\sqrt{10}$.
32. a.
33. $a^2\sqrt{3}$.
34. $2c^2$.
35. $\frac{a}{b}$.
36. $\frac{1}{2a}\sqrt{2}$.
37. $3a^2$.
38. $2n\sqrt[3]{2}$.
39. $7x\sqrt[3]{x^2}$.
40. $2y$.
41. $\frac{1}{x^2}\sqrt[3]{ax^2}$.
42. $5a\sqrt[4]{a}$.
43. $\frac{3}{2y}$.
44. $\frac{3a}{2y^2}\sqrt[4]{12y^3}$.
45. $\frac{2}{5}\sqrt{5m}$.
46. $\sqrt{a}+\sqrt{b}$.
47. $1+\sqrt{a}$.
48. $\sqrt{b}+1$.
49. $x-y$.
50. $ax\sqrt{x}-b\sqrt{y}$.
51. $cy\sqrt[3]{y^2}-d\sqrt[3]{z}$.
52. 3.
53. $\frac{1}{3}\sqrt[4]{27}$.
54. $\sqrt[6]{7}$.
55. $\sqrt[6]{24}$.
56. $\sqrt[6]{\frac{2}{3}}$.
57. 0.6.
58. 0.2.
59. 0.3.
60. $\frac{4}{15}$.
61. $\frac{1}{5}\sqrt{10}$.
62. $\frac{1}{2}\sqrt[5]{16}$.
63. $\sqrt{3}$.
64. 7.
65. $\sqrt{5}$.
66. $2\sqrt{3}$.
67. $\frac{1}{2}\sqrt{6}$.
68. $\frac{2}{3}\sqrt{2}$.
69. $\frac{1}{3}\sqrt{7}$.
70. $\frac{1}{7}\sqrt{210}$.
71. $9\sqrt{3}+15$.
72. $6\sqrt{2}+8$.
73. $\frac{1}{a}\sqrt[6]{a^5}$.

74. $\sqrt[15]{b}$.
75. $\frac{1}{m}\sqrt[12]{m^{11}}$.
76. $\sqrt[18]{x^{11}}$.
77. $\frac{1}{x^2}\sqrt[6]{4x^5}$.
78. $\sqrt[35]{\frac{y^{13}}{32}}$.
79. $\frac{1}{6z}\sqrt{2}$.
80. $\frac{1}{a}\sqrt[77]{a^{68}b^{30}}$.
81. $\sqrt[42]{a^5c^{10}}$.
82. $a\sqrt{a}$.
83. $b^2\sqrt{b}$.
84. $z^4\sqrt{z}$.
85. $\frac{y^2}{2}\sqrt{6y}$.
86. $\frac{1}{5x^2}\sqrt[3]{25mx}$.
87. $\frac{1}{2x^2y^3}\sqrt[4]{8x^3y^2}$.
88. $\frac{2a}{5x^2}\sqrt[3]{5a^2x^2}$.
89. $\frac{3}{x-y}\sqrt{x^2-y^2}$
90. $\frac{1}{2}\sqrt[4]{m^3}+\frac{3}{8}\sqrt[4]{mn^2}$.
91. $\frac{1}{5}$.
92. 4.
93. $\frac{5}{2}$.
94. $\frac{1}{6}\sqrt{6}$.
95. 17.
96. $\frac{1}{6}$.
97. 4.
98. $\frac{1}{8}$.
99. $\frac{1}{a}\sqrt{a}$.
100. $\frac{1}{m^2}$.
101. $\frac{1}{a}$.
102. $\frac{n^3}{m^3}\sqrt[3]{m^2}$.
103. $\frac{b^2}{a^2}\sqrt{ab}$.
104. a.
105. $m\sqrt[15]{m}$.
106. $\frac{1}{x^6}\sqrt[6]{x^5}$.
107. $\frac{1}{a^2}\sqrt[10]{a^9}$.
108. a^4.
109. $\frac{1}{m^3}\sqrt[3]{m^2}$.
110. $\sqrt[6]{a^5}$.
111. $x^2\sqrt[10]{x^7}$.
112. $\frac{1}{a}\sqrt[15]{a^{14}}$.
113. $4\sqrt[3]{4}$.
114. $24\sqrt{3}$.
115. $\sqrt[3]{9}$.

Ex. 47. Page 159.

1. a^3.
2. m^2.
3. n^3.
4. x^3.
5. y^2.
6. ab^2.
7. $m^7\sqrt{m}$.
8. $n^2\sqrt[3]{n^2}$.

9. $y\sqrt[4]{y}$.

10. $b^2c\sqrt[3]{b^2c}$.

11. $xz^2\sqrt[8]{x^6z^5}$.

12. $a^2\sqrt[3]{4a^2}$.

13. $1\frac{3}{5}$.

14. $3a^2$.

15. a^2c.

16. $2a^4\sqrt[4]{a}$.

17. a^2x^3.

18. $(x+y)^6$.

19. $(x-y)\sqrt[5]{(x-y)^3}$.

20. $(a^2-b^2)^2$.

21. $\sqrt[4]{(1+a^2)^9}$.

22. $(x^2-y^2)^{2n}$.

23. $a\sqrt[3]{ab^2}+m^3+x\sqrt{ax}-2m\sqrt[6]{a^4b^2m^3}+2\sqrt[12]{a^{11}b^4x^9}$
$-2m\sqrt[4]{m^2ax^3}$.

24. $\sqrt[x]{a^4}+\sqrt[y]{b^2}-2\sqrt[xy]{a^{2y}b^x}$.

25. $\sqrt[x]{m^2}+\sqrt[y]{n^2}+\sqrt[z]{p^4}-2\sqrt[xy]{m^yn^x}+2\sqrt[xz]{m^zp^{2x}}-2\sqrt[yz]{n^zp^{2y}}$.

26. $2a^3b$.

27. $-xy^2$.

28. $(x^2-y^2)^2$.

29. $-(a^2-b^2)^3$.

30. 5.

31. 16.

32. 127.

33. -131.

34. $9\sqrt[3]{9}$.

35. $-4\sqrt[4]{2}$.

36. 169.

37. 6.

38. $\sqrt{n}$.

39. v^2.

40. x^{12}.

41. x^3.

42. y^{20}.

43. $\sqrt[3]{a}$.

44. $\sqrt[6]{b}$.

45. $\sqrt[12]{c}$.

46. $\sqrt[30]{2m}$.

47. $\sqrt[8]{3a^2}$.

48. $\sqrt[3]{a}$.

49. $\sqrt{m}$.

50. $\sqrt[3]{y}$.

51. $\sqrt[3]{a^2}$.

52. $\sqrt[12]{m^5}$.

53. $\sqrt{5}$.

54. $\sqrt{3}$.

55. $x-y$.

56. $(a-b)^2$.

57. $(m+n)^2$.

58. $\sqrt[3]{(3a-2b)^2}$.

59. $(1-x^2)^6$.

60. $\sqrt[3]{2x^2}$.

61. $a^3\sqrt[3]{2}$.

62. $m\sqrt[3]{n^2}$.

63. $m\sqrt[13]{6}$.

64. $2a^2\sqrt[7]{3}$.

Ex. 48. Page 160.

1. $\frac{2(3+\sqrt{6})}{3}$.

2. $\frac{21(4+\sqrt{6})}{10}$.

3. $4(5\sqrt{3}-3\sqrt{5})$.

4. $\frac{3(8\sqrt{7}-7\sqrt{2})}{2}$.

5. $40(3\sqrt{2}+\sqrt{3})$.

6. $4-\sqrt{15}$.

7. $\frac{8+\sqrt{55}}{3}$.

8. $\frac{23-2\sqrt{130}}{3}$.

9. $\frac{11+3\sqrt{14}}{5}$.

10. $4+\sqrt{15}$.

11. $-\frac{12+9\sqrt{2}+20\sqrt{3}+15\sqrt{6}}{2}$.

12. $\frac{3-\sqrt{3}-12\sqrt{5}+4\sqrt{15}}{12}$.

13. $\frac{7(4\sqrt{5}-5)+15(\sqrt{10}-4\sqrt{2})}{55}$.

14. $\frac{5(2\sqrt{3}+\sqrt{6})-6(1+\sqrt{2})}{57}$

15. $-\frac{7(2+\sqrt{11})+3(2\sqrt{7}+\sqrt{77})}{42}$.

16. $\frac{18(120\sqrt{6}+205\sqrt{3}-219\sqrt{2}-308)}{529}$.

17. $\frac{\sqrt{2}+\sqrt{6}}{2}$. **18.** $\sqrt{3}+\sqrt{2}$. **19.** $2\sqrt{2}+\sqrt{5}-\sqrt{10}-2$.

20. $\frac{195-75\sqrt{3}-187\sqrt{2}+137\sqrt{6}}{94}$.

21. $\frac{1}{2}(19\sqrt{2}+16\sqrt{3}-10\sqrt{5}-10\sqrt{6}+8\sqrt{10}+8\sqrt{15}$
$-5\sqrt{30}-22)$.

Ex. 49. Page 161.

1. $2\sqrt{-1}$. **3.** $9\sqrt{-1}$. **5.** $a\sqrt{-1}$.

2. $5\sqrt{-1}$. **4.** $12\sqrt{-1}$. **6.** $b^2\sqrt{-1}$.

7. $2\sqrt[4]{-1}=\sqrt{2}\,(1+\sqrt{-1})$.

8. $3\sqrt[4]{-1}=\frac{3}{2}\sqrt{2}\,(1+\sqrt{-1})$.

9. $5\sqrt[4]{-1}=\frac{5}{2}\sqrt{2}\,(1+\sqrt{-1})$.

10. $2\sqrt[6]{-1}=2\sqrt{-1}$.

11. $3\sqrt[6]{-1}=3\sqrt{-1}$.

12. $2\sqrt[8]{-1}=\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\sqrt{-1}$.

13. $x^2\sqrt[4]{-1}=\frac{x^2}{2}\sqrt{2}\,(1+\sqrt{-1})$.

14. $y^2\sqrt[4]{y}\sqrt[4]{-1}=y^2\sqrt{\frac{y}{4}}\,(1+\sqrt{-1})$.

15. $z^3\sqrt{-1}$. **16.** $u^2\sqrt{-1}$. **17.** $\frac{1}{2}\sqrt{-1}$.

18. $\frac{a^2}{b}\sqrt{-1}$. **19.** $\sqrt{x^2+y^2\sqrt{-1}}$.

20. $\sqrt{x^2+y^2}\sqrt[4]{-1}=\sqrt{\frac{x^2+y^2}{2}}(1+\sqrt{-1})$.

21. $(3x-2y^2)^2\sqrt{-1}$.

22. $3x^3\sqrt{-1}$. **23.** $9m^3\sqrt{-1}$.

24. $-1;\ 1;\ 1;\ 1;\ \sqrt{-1};\ -\sqrt{-1};\ -\sqrt{-1}$.

25. $-\sqrt{-1};\ -\sqrt{-1};\ 1;\ 1;\ -1;\ \sqrt{-1};\ -1$.

26. $10\sqrt{-1}$. **28.** $2(x-y+4xy)\sqrt{-1}$.

27. $(3a^2-7a)\sqrt{-1}$. **29.** $(10a^3-a^4)\sqrt{-1}$.

30. $m(1+2n)i$.

31. $11-3i$.

32. $\frac{25a}{2}-\frac{33b}{2}\sqrt{-1}$.

33. $2[13+(1+b)i]$.

34. $-a$.

35. -12.

36. $-\sqrt{mn}$.

37. $-\sqrt{y}$.

38. $35\sqrt{-1}$.

39. $a^6\sqrt{-1}$.

40. $-12yz^3$.

41. $3m^3$.

42. $-z$.

43. -30.

44. $8\sqrt{14}$.

45. 77.

46. $-\sqrt{a^2-b^2}$.

47. $(x-y)\sqrt{-1}$.

48. $(y-5)\sqrt{-1}$.

49. $-3\sqrt{2-y}$.

50. a^2+1.

51. m^2+b^2.

52. $47-\sqrt{-1}$.

53. $xy+2b+(2y-bx)\sqrt{-1}$.

54. $6(xy+4)+(9x-16y)\sqrt{-1}$.

55. $2ac+3bd+(3bc-2ad)\sqrt{-1}$.

56. $16-2i\sqrt{2}$.

57. $mn+12\sqrt{bc}+(4m\sqrt{c}-3n\sqrt{b})i$.

58. $2(\sqrt{15}+2\sqrt{3})+2(3-\sqrt{10})i$.

59. $-a\sqrt{-1}$.

60. $-\sqrt{-1}$.

61. $-\frac{di}{2}$.

62. $\sqrt{3}$.

63. $\frac{1}{n}\sqrt{mn}$.

64. $\sqrt{a}$.

65. $-i\sqrt{x}$.

66. $-\sqrt{-1}$.

67. $-i\sqrt{m}$.

68. $\sqrt{x}$.

69. 2.

70. $m\sqrt{2}$.

Ex. 50. Page 163.

1. 2.
2. 3.
3. $3\frac{1}{2}$.
4. $12\frac{1}{4}$.
5. $\frac{1}{3}$.
6. $\frac{1}{4}$.
7. $\frac{4}{9}$.
8. $-\frac{1}{3}$.
9. $\frac{1}{20}$.
10. 2.
11. -12.
12. $-\frac{1}{3}$.
13. $6\frac{2}{3}$.
14. $-2\frac{1}{2}$.
15. -6.
16. $3\frac{1}{3}$.
17. 5.
18. -5.
19. $1\frac{3}{5}$.
20. -5.
21. 1.
22. 4.
23. $\frac{7}{9}$.
24. $2\frac{1}{2}$.
25. $\frac{1}{2}$.
26. 2.
27. $-\frac{5}{7}$.
28. -3.
29. 1.
30. $\frac{1}{4}$.
31. 49.
32. 5.
33. 23.
34. 8.
35. 6.
36. 5.
37. 49.
38. 4.
39. 4.
40. $8\frac{1}{2}$.
41. 60.
42. $4\frac{1}{2}$.
43. $\frac{1}{9}$.
44. $0;\ -\frac{1}{4}$.
45. $\frac{1}{3}$.
46. 25.
47. $\frac{3}{4}$.
48. $0;\ \frac{1}{3}$.
49. $\frac{1}{3}$.
50. $200\frac{1}{2}$.
51. 4.
52. 4.
53. $\frac{a}{a+2\sqrt{a}}$.
54. $\sqrt{3};\ 0$.
55. $\pm\frac{1}{2}\sqrt{3}$.
56. ± 3.
57. $6;\ 2$.
58. $7;\ 3$.
59. $4;\ -25$.
60. $196;\ 49$.
61. $1\frac{8}{15};\ \frac{2}{3}$.
62. $2;\ \frac{1}{2}$.
63. $5;\ -2$.
64. $6\frac{9}{16};\ 6$.
65. $7;\ 0$.
66. $2;\ -\frac{61}{90}$.
67. $6;\ \frac{14}{5}$.
68. $5;\ \frac{5}{3}$.
69. $2;\ -\frac{10}{11}$.
70. $9;\ 1$.
71. $4;\ \frac{8}{9}$.
72. $3;\ -\frac{11}{8}$.
73. $5;\ 1$.
74. $\frac{1}{2};\ -\frac{9}{10}$.
75. $8;\ 4\frac{4}{17}$.
76. $4;\ -\frac{4}{7}$.
77. $\frac{1}{2};\ -4\frac{1}{6}$.
78. $\frac{5}{4};\ 55\frac{1}{4}$.
79. $4;\ -\frac{8}{9}$.
80. $-4;\ -7$.
81. $3;\ \frac{9}{10}$.
82. $2;\ -16\frac{1}{2}$.
83. $1;\ -\frac{11}{24}$.
84. $3;\ -3\frac{5}{7}$.
85. $7;\ -6$.
86. $\frac{1}{2};\ -\frac{5}{7}$.
87. $1;\ -\frac{1}{3}$.
88. $5;\ -4\frac{11}{13}$.
89. $3;\ -\frac{2}{3}$.
90. ± 2.
91. $\pm\frac{1}{2}$.
92. $\frac{9}{5};\ \frac{1}{5}$.
93. $4;\ \frac{1}{2}$.
94. ± 2.
95. $\pm\frac{2}{3}$.
96. $4;\ -\frac{1}{4}$.
97. $\frac{b}{a};\ -1$.
98. $\frac{4a}{3};\ \frac{a}{3}$.

99. $\frac{1}{2a}(1 \pm \sqrt{1+4b^2})$.

100. $\frac{1}{2}(\sqrt{a} \pm \sqrt{b})$.

101. $\pm \frac{a\sqrt{3}}{2}$.

102. $\frac{2a}{3}; \ -2a$.

103. $4a; \ -2a$.

104. $\frac{a}{3}; \ -\frac{a+2b}{3}$.

105. $\frac{a(2 \pm \sqrt{2})}{2}$.

106. $\pm \frac{2a}{5}\sqrt{5}$.

107. $b \pm \sqrt{b^2 - ab}$.

108. n.

109. $\pm 2b$.

110. $\pm \frac{a}{2}\sqrt{\frac{a^2-4}{a^2-1}}$.

111. $0; \ \frac{9a}{16}$.

112. $\pm a\sqrt{\frac{a}{a-1}}$.

113. $\frac{a}{2b^2}; \ -\frac{9a}{2b^2}$.

114. $0; \ a$.

115. $\pm \sqrt{ab}$.

116. $\frac{a}{1+\sqrt[3]{a^2}}$.

117. $\frac{a+b}{2} \pm \frac{a-b}{4}\sqrt{3}$.

118. $\pm 2a$.

119. $a; \ b$.

120. $\frac{a-b}{2} \pm \frac{(a+b)c}{2\sqrt{c^2+4}}$.

121. $\frac{a+b}{2} \pm \frac{(a-b)c}{2\sqrt{c^2-4}}$.

122. $\frac{a+b}{2} \pm \frac{a-b}{2c}\sqrt{c^2+4}$.

123. $x = 6a$, or $-\frac{282a}{7}$.

Ex. 51. Page 169.

1. \$3900.48.
2. \$13,823.13.
3. \$4855.87.
4. \$35,705.48.
5. \$1377.01.
6. \$8,646,290.
7. \$6250.
8. \$368,429.
9. \$124,225.88.
10. \$5643.20.
11. \$3129.29.
12. \$606.77.
13. \$1459.52.

14. First, $2368.39; second, $2377.52; third, $2382.18; fourth, $2385.34.

15. $7800.

16. $1400.

17. $4315.

18. $955.025.

19. $831.04.

20. $1614.97.

21. 8 yrs.

22. 11 yrs.

23. About $7\frac{1}{3}$ yrs.

24. About $54\frac{1}{2}$ yrs.

25. First, 23.45 yrs.; second, 20.15 yrs.; third, 17.67 yrs.; fourth, $14\frac{1}{3}$ yrs., nearly.

26. 4%.

27. $4\frac{3}{4}$%.

28. $4\frac{1}{2}$%.

29. 3%.

30. 6%.

31. $1260.12.

32. $3916.80.

33. 21.64%.

34. $110.28.

35. 3%.

36. 67.826 gals.

37. $74,272.88^{\text{cbm}}$.

38. 4.651%.

39. $2812.83.

40. $51,280 and $52,700.

41. $5097.97 and $5490.97.

42. 100 yrs.

43. The $35,000 of B, discounted at 5%, is reduced to $30,234.31; the $33,000 of C to $31,453.52. Then C has offered $1453.52 more than A, and $1219.21 more than B.

44. $10,297.16.

45. 20 yrs.

46. 24 yrs.

47. $23,569.63.

48. $12,144.28.

49. 1,111,282 inhabitants.

50. $66,868.

51. $76,080.

52. 1979.62^{m}.

53. 13.58 yrs.

54. 21.26 yrs.

55. 11 yrs.

56. $12,500.

57. $16,431.95.

58. 17 yrs.

59. $2200.50.

60. $105,078.50.

61. $5000.

62. $4\frac{1}{2}$%.

Ex. 52. Page 176.

1. 4 : 1.

2. 3 : 2.

3. 1 : 7.

4. 1 : 10.

5. 9 : 1.

6. 7 : 8 : 9 : 10.

7. 11 : 9.

8. 13 : 16.

9. 6 : 5.

10. 5 : 9.

11. 3 : 10.

12. 9 : 16.

13. 8 : 27.

14. $\frac{m}{t} : \frac{n}{p}$; or $mp : nt$; 14 : 15.

15. $\frac{18}{27} = \frac{50}{75}$.

16. $\frac{285}{969} = \frac{195}{663}$.

17. 150.

18. $3\frac{3}{4}$.

19. $9\frac{1}{3}$.

20. $1\frac{1}{2}$.

21. $11\frac{1}{3}$.

22. $11\frac{1}{25}$.

23. 14.

24. 18.
25. $6\frac{1}{6}$.
26. $1\frac{3}{7}$.
27. $3n$.
28. $\frac{1}{3}$ qt.
29. $\frac{6bpq}{a}$.
30. $4\frac{1}{3}$.
31. $2\frac{2}{3}$.
32. $\frac{4}{9}$.
33. 1.
34. $(p+q)^2$.
35. $\frac{1}{m-n}$.
36. $\frac{a^2(a-b)}{c}$.
37. $\frac{1-b}{a}$.
38. 1.
39. 12.
40. $4\frac{3}{8}$.
41. $45\frac{1}{3}$.
42. $\frac{np}{m}$.
43. $\frac{a}{bc}$.
44. $\frac{m(m-n)}{n(m+n)}$.
45. $\frac{1-n}{m}$.
46. $\frac{a^2+b^2}{ab}$.
47. 4.
48. 9.
49. 25.
50. $10\frac{1}{3}$.
51. $\frac{n^2}{m}$.
52. $\frac{p-q}{r(p+q)}$.
53. $\frac{4p^3}{(m^2-mp+p^2)^2}$.
54. $\frac{(l-m)^3}{(l+m)^3}$.
55. $\frac{4(a-b)^4}{9b^4(a+b)^2}$.
56. 9.
57. 6.
58. 10.
59. 5.
60. 7.
61. m.
62. $\frac{q}{2p}$.
63. 1.
64. $c+d$.

Ex. 53. Page 179.

1. $l=55$,
$s=403$.

2. $l=38$,
$s=258$.

3. $l=174\frac{1}{2}$,
$s=2625$.

4. $l=20.9$,
$s=2190$.

5. $l=10\frac{4}{7}$,
$s=83\frac{4}{7}$.

6. $l=28$,
$s=364$.

7. $l=73n$,
$s=1368n$.

8. $l=2m+52n$,
$s=28m+364n$.

9. $l=1+30m$,
$s=11+165m$.

10. $d=11$,
$s=884$.

11. $d=4$,
$s=5100$.

12. $d=\frac{3}{4}$,
$s=2757\frac{1}{4}$.

13. $d=-7$,
$s=2124$.

14. $d=-0.3$,
$s=63$.

15. $d=7a^3$,
$s=1575a^3$.

16. $d = \frac{1}{2}a^3$; $s = 70\,b^2 + 297\frac{1}{2}a^3$.

17. 20, 27, 34, etc.

18. $1\frac{1}{16}$, $1\frac{2}{16}$, $1\frac{3}{16}$, etc.

19. $\frac{19}{36}$, $\frac{20}{36}$, $\frac{21}{36}$, etc.

20. 42, 37, 32, etc.

21. $a = 2$,
$s = 1661$.

22. $a = 11$,
$s = 1518$.

23. $a = \frac{5}{6}$,
$s = 272\frac{1}{3}$.

24. $a = 56$,
$s = 680$.

25. $a = 65$,
$s = 455$.

26. $n = 38$,
$s = 6973$.

27. $n = 13$,
$s = 286$.

28. $n = 94$,
$s = 5463\frac{3}{4}$.

29. $d = 1\frac{1}{2}$,
$n = 36$.

30. $d = 4$,
$n = 40$.

31. $d = 5$,
$n = 18$.

32. $d = 4$,
$n = 11$.

33. $d = 10$,
$l = 140$.

34. $d = 1\frac{1}{2}$,
$l = 29\frac{1}{2}$.

35. $d = \frac{2}{3}x$,
$l = \frac{58}{3}x$.

36. $d = -4$,
$l = -16$.

37. $a = 9$,
$d = 2$.

38. $a = 18$,
$d = 5$.

39. $a = -\frac{1}{3}$,
$d = \frac{1}{3}$.

40. $a = 0$,
$d = 7$.

41. $a = 17$,
$l = 97$.

42. $a = 11$,
$l = 88$.

43. $a = -9$,
$l = 3$.

44. $a = 11$,
$l = -13$.

45. $n = 15$,
$l = 108$.

46. $n = 16$,
$l = 69$.

47. $n = 62$,
$l = 50.3$.

48. $n = 22$.
$l = 144$.

49. $n = 14$,
$a = 12$.

50. $n = 21$ (or 24),
$a = 8$ (or -4).

51. $n = 32$ (or 41),
$a = 2\frac{1}{7}$ (or $-1\frac{5}{7}$).

52. $n = 16$,
$a = 315$.

53. 719.5^{m}.

54. No; he would pay \$10.24 instead of \$8.

55. 3825 ft.

56. 3200 ft.

57. 406.7^{m}.

58. $10{,}255.7^{\text{m}}$; 46 sec. **59.** 4.1156 sec. **60.** 3, 10, 17, 24, 31.
61. 4, 11, 18. **62.** 4, 7, 10, 13, 16.
63. $-8, -5, -2, 1, 4, 7, 10, 13, 16, 19, 22$.

Ex. 54. Page 183.

1. 819.
2. 22,140.
3. 51,039.
4. 338,350.
5. 658,875.
6. 10,468,290.
7. 333,833,500.
8. 2,646,275.
9. 14,400.
10. 608,400.
11. 178,200.
12. 1,589,525.
13. 16,523,009.
14. 396,459.
15. 91.
16. 285.
17. 140.
18. 1785.
19. 3311.
20. 6930.
21. 476.
22. 271.
23. 960.
24. 1581.
25. 2079.
26. 1441.
27. 56.
28. 364.
29. 969.
30. 220.
31. 1330.
32. 2925.
33. 525.
34. 605.
35. 1784.
36. 1085.
37. 145.
38. 80.
39. 1547.
40. 2856.
41. 4750.
42. 5428.
43. 1030.
44. 1757.
45. 912.
46. 3490.

Ex. 55. Page 185.

1. $l = 256$, $S = 508$.
2. $l = 13{,}122$, $S = 19{,}682$.
3. $l = 4096$, $S = 5461$.
4. $l = \frac{1}{6561}$, $S = 13\frac{3280}{6561}$.
5. $l = 0.00013$, $S = 14.44443$.
6. $l = \frac{7}{256}$, $S = 111\frac{249}{256}$.
7. $l = \frac{1}{512}$, $S = 1\frac{511}{512}$.
8. $l = 7.59375$, $S = 20.781250$.
9. $l = 0$, $S = 2$.
10. $l = 0$, $S = 10\frac{2}{3}$.
11. $l = 0$, $S = 2\frac{1}{3}$.
12. $l = 0$, $S = 12\frac{1}{2}$.
13. $l = 0$, $S = 1\frac{1}{3}$.
14. $l = m^3n^6$,

$$S = \frac{m^3}{n-1}(n^7 - 1).$$

15. $l=a(1+x)^7$,
$S=\frac{a}{x}[(1+x)^8-1]$.

16. $l=\frac{a}{(a^2-1)^9}$,
$S=\frac{a[(a^2-1)^{10}-1]}{(a^2-2)(a^2-1)^9}$.

17. $l=\frac{b}{(1-m)^8}$,
$S=\frac{b[1-(1-m)^9]}{m(1-m)^8}$.

18. $l=b$,
$S=\frac{b}{x}[(1+x)^n-1]$.

19. $r=6$,
$S=12{,}440$.

20. $r=5$,
$S=39{,}062$.

21. $r=\frac{1}{3}$,
$S=53\frac{79}{81}$.

22. $r=4$,
$S=65{,}535$.

23. $r=\frac{1}{4}$,
$S=63\frac{63}{64}$.

24. $r=2$,
$S=7161$.

25. $r=7$.

26. $r=3$.

27. $r=2$.

28. $r=6$.

29. $r=\frac{2}{3}$.

30. $r=\frac{5}{2}$.

31. $r=2$.

32. $r=3$.

33. $r=\frac{1}{5}$.

34. $a=2$,
$S=254$.

35. $a=1$,
$S=97{,}656$.

36. $a=3$,
$S=36{,}621{,}093$.

37. $a=2$,
$S=53\frac{2039}{2187}$.

38. $a=56$,
$S=89\frac{148101}{262144}$.

39. $a=6$,
$S=8\frac{26}{27}$.

40. $S=4599$,
$n=9$.

41. $S=126$,
$n=6$.

42. $S=381(1+\sqrt{2})$,
$n=14$.

43. $S=80\frac{242}{243}$,
$n=9$.

44. $S=24\frac{17}{27}$,
$n=6$.

45. $r=3$,
$n=7$.

46. $r=7$,
$n=5$.

47. $r=\frac{1}{2}$,
$n=6$.

48. $r=\frac{1}{5}$,
$n=5$.

49. $n=9$,
$l=19{,}683$.

50. $n=9$,
$l=2048$.

51. $a=5$,
$l=320$.

52. $a=4$,
$l=131{,}072$.

53. $a=1$,
$n=5$.

54. $a=2$,
$n=8$.

55. $r=7$ or -8,
$l=882$ or 1152.

56. 239.999999 gals.

57. $\frac{8}{11}$.

58. Ratio = 4 or $\frac{1}{4}$; first term = 3 or 192.

59. $-871,696,100$. **60.** 2730. **61.** $6\frac{3}{4}$.

Ex. 56. Page 188.

1. 403,291,461,126,605,635,584,000,000 ways.
2. 87,178,291,200 ways.
3. 265,252,859,812,191,058,636,308,480,000,000 ways.
4. 40,320 ways.
5. 15,511,210,043,330,985,984,000,000 ways.
6. 2,432,902,008,176,640,000 ways.
7. 362,880 numbers.
8. 3,265,920 numbers.
9. 1,060,137,318,240 ways.
10. 2,331,418,652,362,800 ways.
11. 12.
12. 7920.
13. 24,504,480.
14. 2,095,133,040.
15. 720 words.
16. 4320 words.
17. 4 *a*'s.
18. 132.
19. 235,989,936,000.
20. 56 ways.
21. 3,603,600 arrangem'ts.
22. 358,800; 165,765,600.
23. 729, allowing repetitions.
24. 900 numbers; 9000 numbers.
25. 205,200 words.
26. 2,052,000 words.
27. 11,400 words.
28. 5040 ways; 30,240 ways.
29. $n = 23$.
30. $m = 17$.
31. 256 numbers; 6561 numbers.
32. 248,832.
33. 56,874,039,553,216 arrangements.
34. 19,676,527,011,956,855,056.
35. 146,813,779,479,484.
36. $n = 14$.
37. $p = 6$.
38. 220.
39. 2,558,620,845.
40. 56.
41. 495.
42. 121,399,651,100.
43. 30 divisors.
44. 12,672,800 ways.
45. 56 ways.
46. 455 ways.
47. 280 costumes.
48. 768,625.

49. 187,467,280.

50. 27,405 nights; each soldier will serve 3654 times.

51. 252 combinations; a will enter in 126; a, b, c will together enter in 21 combinations.

52. Let C_{n-1} be the number of combinations of m objects taken $n-1$ at a time; to find the number of the combinations taken n at a time, C_{n-1} must be multiplied by

$$\frac{m-n+1}{n}=\frac{m+1}{n}-1.$$

The product will increase so much that we shall have $\frac{m+1}{n}-1>1$, or $\frac{m+1}{n}>2$, or $n<\frac{m+1}{2}$. If m is even, $n=\frac{m}{2}$; if m is odd, $n=\frac{1}{2}(m-1)$ or $\frac{1}{2}(m+1)$.

53. 10.

54. 17.

55. 238 divisors without counting the number itself.

56. 94 divisors.

57. 718 divisors.

58. Combinations n at a time $=\frac{m(m-1)\cdots\cdots(m-n+1)}{1\times 2\times\cdots\cdots\times n}$.

Combinations $m-n$ at a time

$$=\frac{m(m-1)\cdots\cdots[m-(m-n)+1]}{1\times 2\times\ldots\ldots\times(m-n)};\text{ whence,}$$

$$\frac{\text{combinations } n \text{ at a time}}{\text{combinations } m-n \text{ at a time}}$$

$$=\frac{m(m-1)\cdots\cdots(m-n+1)}{1\times 2\times 3\times\cdots\cdots\times n}\times\frac{1\times 2\times 3\cdots\cdots(m-n)}{m(m-1)\cdots\cdots(n+1)}$$

$$=\frac{1\times 2\times 3\cdots\cdots(m-n)(m-n+1)\cdots\cdots(m-1)m}{1\times 2\times 3\cdots\cdots n(n+1)\cdots\cdots(m-1)m}$$

$$=\frac{\underline{|m}}{\underline{|m}}=1;$$

then the number of combinations n at a time equals the number of combinations $m-n$ at a time.

59. 635,013,559,600 combinations, and will take 482,936.77 years to form them.

60. 1,528,562,927,268,877,500 combinations.

61. 704,982,460,000 ways.

Ex. 57. Page 195.

1. $a^3 + 3a^2x + 3ax^2 + x^3$.

2. $b^4 + 4b^3x + 6b^2x^2 + 4bx^3 + x^4$.

3. $d^5 + 5d^4y + 10d^3y^2 + 10d^2y^3 + 5dy^4 + y^5$.

4. $c^6 + 6c^5z + 15c^4z^2 + 20c^3z^3 + 15c^2z^4 + 6cz^5 + z^6$.

5. $a^7 + 7a^6p + 21a^5p^2 + 35a^4p^3 + 35a^3p^4 + 21a^2p^5 + 7ap^6 + p^7$.

6. $m^{11} + 11m^{10}n + 55m^9n^2 + 165m^8n^3 + 330m^7n^4 + 462m^6n^5$
$+ 462m^5n^6 + 330m^4n^7 + 165m^3n^8 + 55m^2n^9$
$+ 11mn^{10} + n^{11}$.

7. $p^{13} + 13p^{12}v + 78p^{11}v^2 + 286p^{10}v^3 + 715p^9v^4 + 1287p^8v^5$
$+ 1716p^7v^6 + 1716p^6v^7 + 1287p^5v^8 + 715p^4v^9$
$+ 286p^3v^{10} + 78p^2v^{11} + 13pv^{12} + v^{13}$.

8. $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$.

9. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$.

10. $a^7 - 7a^6x + 21a^5x^2 - 35a^4x^3 + 35a^3x^4 - 21a^2x^5$
$+ 7ax^6 - x^7$.

11. $h^{13} + 13h^{12}z + 78h^{11}z^2 + 286h^{10}z^3 + 715h^9z^4 + 1287h^8z^5$
$+ 1716h^7z^6 + 1716h^6z^7 + 1287h^5z^8 + 715h^4z^9$
$+ 286h^3z^{10} + 78h^2z^{11} + 13hz^{12} + z^{13}$.

12. $h^{11} - 11h^{10}z + 55h^9z^2 - 165h^8z^3 + 330h^7z^4 - 462h^6z^5$
$+ 462h^5z^6 - 330h^4z^7 + 165h^3z^8 - 55h^2z^9 + 11hz^{10} - z^{11}$.

13. $a^8 - 8a^7c + 28a^6c^2 - 56a^5c^3 + 70a^4c^4 - 56a^3c^5 + 28a^2c^6$
$- 8ac^7 + c^8$.

14. $b^9 - 9b^8p + 36b^7p^2 - 84b^6p^3 + 126b^5p^4 - 126b^4p^5 + 84b^3p^6 - 36b^2p^7 + 9bp^8 - p^9.$

15. $z^{13} - 13z^{12}d + 78z^{11}d^2 - 286z^{10}d^3 + 715z^9d^4 - 1287z^8d^5 + 1716z^7d^6 - 1716z^6d^7 + 1287z^5d^8 - 715z^4d^9 + 286z^3d^{10} - 78z^2d^{11} + 13zd^{12} - d^{13}.$

16. $1 + 12x + 66x^2 + 220x^3 + 495x^4 + 792x^5 + 924x^6 + 792x^7 + 495x^8 + 220x^9 + 66x^{10} + 12x^{11} + x^{12}.$

17. $1 - 9x + 36x^2 - 84x^3 + 126x^4 - 126x^5 + 84x^6 - 36x^7 + 9x^8 - x^9.$

18. $1 - 8a + 28a^2 - 56a^3 + 70a^4 - 56a^5 + 28a^6 - 8a^7 + a^8.$

19. $1 + 15a + 105a^2 + 455a^3 + 1365a^4 + 3003a^5 + 5005a^6 + 6435a^7 + 6435a^8 + 5005a^9 + 3003a^{10} + 1365a^{11} + 455a^{12} + 105a^{13} + 15a^{14} + a^{15}.$

20. $1 - 14z + 91z^2 - 364z^3 + 1001z^4 - 2002z^5 + 3003z^6 - 3432z^7 + 3003z^8 - 2002z^9 + 1001z^{10} - 364z^{11} + 91z^{12} - 14z^{13} + z^{14}.$

21. $x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1.$

22. $x^{11} - 11x^{10} + 55x^9 - 165x^8 + 330x^7 - 462x^6 + 462x^5 - 330x^4 + 165x^3 - 55x^2 + 11x - 1.$

23. $y^8 - 8y^7 + 28y^6 - 56y^5 + 70y^4 - 56y^3 + 28y^2 - 8y + 1$

24. $z^7 + 7z^6 + 21z^5 + 35z^4 + 35z^3 + 21z^2 + 7z + 1.$

25. $z^8 - 16z^7 + 112z^6 - 448z^5 + 1120z^4 - 1792z^3 + 1792z^2 - 1024z + 256.$

26. $8 + 12x + 6x^2 + x^3.$ **27.** $16 - 32x + 24x^2 - 8x^3 + x^4.$

28. $6561 + 17{,}496x + 20{,}412x^2 + 13{,}608x^3 + 5670x^4 + 1512x^5 + 252x^6 + 24x^7 + x^8.$

29. $x^9 - 27x^8 + 324x^7 - 2268x^6 + 10{,}206x^5 - 30{,}618x^4 + 61{,}236x^3 - 78{,}732x^2 + 59{,}049x - 19{,}683.$

30. $z^6 + 30z^5 + 375z^4 + 2500z^3 + 9375z^2 + 18{,}750z$
$+ 15{,}625.$

31. $a^8 - 40a^7 + 700a^6 - 7000a^5 + 43{,}750a^4 - 175{,}000a^3$
$+ 437{,}500a^2 - 625{,}000a + 390{,}625.$

32. $16{,}384 - 28{,}672b + 21{,}504b^2 - 8960b^3 + 2240b^4$
$- 336b^5 + 28b^6 - b^7.$

33. $h^{10} - 20h^9 + 180h^8 - 960h^7 + 3360h^6 - 8064h^5$
$+ 13{,}440h^4 - 15{,}360h^3 + 11{,}520h^2 - 5120h + 1024.$

34. $a^6 - 60a^5 + 1500a^4 - 20{,}000a^3 + 150{,}000a^2 - 600{,}000a$
$+ 1{,}000{,}000.$

35. $b^5 + 20b^4 + 160b^3 + 640b^2 + 1280b + 1024.$

36. $\frac{1}{32} + \frac{5}{16}x + 1\frac{1}{4}x^2 + 2\frac{1}{2}x^3 + 2\frac{1}{2}x^4 + x^5.$

37. $y^6 + 2y^5 + \frac{5}{3}y^4 + \frac{20}{27}y^3 + \frac{5}{27}y^2 + \frac{2}{81}y + \frac{1}{729}.$

38. $\frac{1}{16384} - \frac{7v}{4096} + \frac{21v^2}{1024} - \frac{35v^3}{256} + \frac{35v^4}{64} - \frac{21v^5}{16} + \frac{7v^6}{4} - v^7.$

39. $a^8 - \frac{8a^7}{5} + \frac{28a^6}{25} - \frac{56a^5}{125} + \frac{14a^4}{125} - \frac{56a^3}{3125} + \frac{28a^2}{15625}$
$- \frac{8a}{78125} + \frac{1}{390625}.$

40. $0.000729 + 0.01458x + 0.1215x^2 + 0.54x^3 + 1.35x^4$
$+ 1.8x^5 + x^6.$

41. $0.0016384 - 0.028672z + 0.21504z^2 - 0.896z^3 + 2.24z^4$
$- 3.36z^5 + 2.8z^6 - z^7.$

42. $335.54432 + 524.288b + 327.68b^2 + 102.4b^3 + 16b^4 + b^5.$

43. $c^4 + 6c^3 + 13.5c^2 + 13.5c + 5.0625.$

44. $a^6 - 12.6a^5 + 66.15a^4 - 185.22a^3 + 291.7215a^2$
$- 245.04606a + 85.766121.$

45. $32a^5 + 80a^4x + 80a^3x^2 + 40a^2x^3 + 10ax^4 + x^5.$

46. $2187a^7 - 5103a^6y + 5103a^5y^2 - 2835a^4y^3 + 945a^3y^4$
$- 189a^2y^5 + 21ay^6 - y^7.$

47. [illegible]

48. [illegible]

49. $\frac{512x^9}{19683} - \frac{256x^8y}{729} - \frac{512x^7y^2}{243} - \frac{1792x^6y^3}{243} - \frac{448x^5y^4}{27}$
$- \frac{[illegible]}{3} - \frac{[illegible]}{9} -$ [illegible] $- 6xy^8 + y^9.$

50. $\frac{177147a^{11}}{4194304} - \frac{649539a^{10}b}{1048576} - \frac{1082565a^9b^2}{262144} - \frac{1082565a^8b^3}{65536}$
$- \frac{360855a^7b^4}{8192} - \frac{168399a^6b^5}{2048} + \frac{56133a^5b^6}{512}$
$- \frac{13365a^4b^7}{128} + \frac{4455a^3b^8}{64} - \frac{495a^2b^9}{16} + \frac{33ab^{10}}{4} - b^{11}.$

51. $x^4 - \frac{12x^3}{5} + \frac{54x^2}{25} - \frac{108x}{125} + \frac{81}{625}.$

52. $x^7 + 14x^6a - 84x^5a^2 + 280x^4a^3 - 560x^3a^4 + 672x^2a^5$
$- 448xa^6 + 128a^7.$

53. $6561x^8 - 17{,}496x^7b + 20{,}412x^6b^2 - 13{,}608x^5b^3$
$+ 5670x^4b^4 - 1512x^3b^5 + 252x^2b^6 - 24xb^7 + b^8.$

54. $y^9 + 18y^8c - 144y^7c^2 + 672y^6c^3 - 2016y^5c^4 + 4032y^4c^5$
$- 5376y^3c^6 + 4608y^2c^7 - 2304yc^8 + 512c^9.$

55. $z^5 + 15z^4a - 90z^3a^2 + 270z^2a^3 - 405za^4 + 243a^5.$

56. $64x^6 + 576x^5a + 2160x^4a^2 + 4320x^3a^3 + 4860x^2a^4$
$+ 2916xa^5 + 729a^6.$

57. $3125d^5 - 9375d^4y + 11{,}250d^3y^2 - 6750d^2y^3 + 2025dy^4$
$- 243y^5.$

58. $16{,}384\,d^7 - 86{,}016\,d^6z + 193{,}536\,d^5z^2 - 241{,}920\,d^4z^3$
$+ 181{,}440\,d^3z^4 - 81{,}648\,d^2z^5 + 20{,}412\,dz^6 - 2187\,z^7.$

59. $0.00032a^5 + 0.0024\,a^4b + 0.0072\,a^3b^2 + 0.0108\,a^2b^3$
$+ 0.0081\,ab^4 + 0.00243\,b^5.$

60. $3.5831808\,b^7 - 4.1803776\,b^6y + 2.0901888\,b^5y^2$
$- 0.580608\,b^4y^3 + 0.096768\,b^3y^4 - 0.0096768\,b^2y^5$
$+ 0.0005376\,by^6 - 0.0000128\,y^7.$

61. $\frac{a^5}{243} + \frac{5a^4b}{162} + \frac{5\,a^3b^2}{54} + \frac{5\,a^2b^3}{36} + \frac{5\,ab^4}{48} + \frac{b^5}{32}.$

62. $\frac{c^8}{390625} - \frac{2\,c^7d}{78125} + \frac{7\,c^6d^2}{62500} - \frac{7\,c^5d^3}{25000} + \frac{7\,c^4d^4}{16000} - \frac{7\,c^3d^5}{16000}$
$+ \frac{7\,c^2d^6}{25600} - \frac{2\,cd^7}{20480} + \frac{d^8}{65536}.$

63. $\frac{512\,m^9}{1953125} + \frac{1728\,m^8p}{390625} + \frac{2592\,m^7p^2}{78125} + \frac{2268\,m^6p^3}{15625}$
$+ \frac{5103\,m^5p^4}{12500} + \frac{15309\,m^4p^5}{20000} + \frac{15309\,m^3p^6}{16000} + \frac{19683m^2p^7}{25600}$
$+ \frac{59049\,mp^8}{163840} + \frac{19683\,p^9}{262144}.$

64. $\frac{64\,z^6}{15625} - \frac{576\,z^5b}{21875} + \frac{432\,z^4b^2}{6125} - \frac{864\,z^3b^3}{8575} + \frac{972\,z^2b^4}{12005}$
$- \frac{2916\,zb^5}{84035} + \frac{729\,b^6}{117649}.$

65. $\frac{2187\,h^7}{78125} - \frac{5103\,h^6v}{31250} + \frac{5103\,h^5v^2}{12500} - \frac{2835\,h^4v^3}{5000} + \frac{945\,h^3v^4}{2000}$
$- \frac{189\,h^2v^5}{800} + \frac{21\,hv^6}{320} - \frac{v^7}{128}.$

66. $a^{10} + 5\,a^8b^3 + 10\,a^6b^6 + 10\,a^4b^9 + 5\,a^2b^{12} + b^{15}.$

67. $x^{18} - 6\,x^{15}a^5 + 15\,x^{12}a^{10} - 20\,x^9a^{15} + 15\,x^6a^{20} - 6\,x^3a^{25} + a^{30}.$

68. $y^{14}+7y^{12}b+21y^{10}b^2+35y^8b^3+35y^6b^4+21y^4b^5$
$+7y^2b^6+b^7.$

69. $d^{24}-8d^{21}b^2+28d^{18}b^4-56d^{15}b^6+70d^{12}b^8-56d^9b^{10}$
$+28d^6b^{12}-8d^3b^{14}+b^{16}.$

70. $a^{27}-9a^{24}y^2+36a^{21}y^4-84a^{18}y^8+126a^{15}y^8-126a^{12}y^{10}$
$+84a^9y^{12}-36a^6y^{14}+9a^3y^{16}-y^{18}.$

71. $d^{24}-24d^{21}x^2+252d^{18}x^4-1512d^{15}x^6+5670d^{12}x^8$
$-13{,}608d^9x^{10}+20{,}412d^6x^{12}-17{,}496d^3x^{14}+6561x^{16}.$

72. $32h^{10}-240h^8x^3+720h^6x^6-1080h^4x^9+810h^2x^{12}$
$-243x^{15}.$

73. $2187a^{14}+25{,}515a^{12}b^3+127{,}575a^{10}b^6+354{,}375a^8b^9$
$+590{,}625a^6b^{12}+590{,}625a^4b^{15}+328{,}125a^2b^{18}$
$+78{,}125b^{21}.$

74. $1-10x^3+40x^6-80x^9+80x^{12}-32x^{15}.$

75. $6561x^{16}-17{,}496x^{14}+20{,}412x^{12}-13{,}608x^{10}+5670x^8$
$-1512x^6+252x^4-24x^2+1.$

76. $1+\frac{16a^5}{3}+\frac{112a^{10}}{9}+\frac{448a^{15}}{27}+\frac{1120a^{20}}{81}+\frac{1792a^{25}}{243}$
$+\frac{1792a^{30}}{729}+\frac{1024a^{35}}{2187}+\frac{256a^{40}}{6561}.$

77. $\frac{p^{25}}{32}+\frac{15p^{20}y^4}{16}+\frac{45p^{15}y^8}{4}+\frac{135p^{10}y^{12}}{2}+\frac{405p^5y^{16}}{2}+243y^{20}.$

78. $\frac{32a^{15}}{243}+\frac{20a^{12}b^2}{27}+\frac{5a^9b^4}{3}+\frac{15a^6b^6}{8}+\frac{135a^3b^8}{128}+\frac{243b^{10}}{1024}.$

79. $0.0016a^{12}-\frac{0.064a^9x^4}{3}+\frac{0.32a^6x^8}{3}-\frac{6.4a^3x^{12}}{27}+\frac{16x^{16}}{81}.$

80. $\frac{78125l^{14}}{2187}+\frac{10937.5l^{12}p^3}{243}+\frac{218.75l^{10}p^6}{9}+\frac{21.875l^8p^9}{3}$
$+1.3125l^6p^{12}+0.14175l^4p^{15}+0.008505l^2p^{18}$
$+0.0002187p^{21}.$

81. $128a^7 - 1344a^6cx + 6048a^5c^2x^2 - 15{,}120a^4c^3x^3$
$+ 22{,}680a^3c^4x^4 - 20{,}412a^2c^5x^5 + 10{,}206ac^6x^6$
$- 2187c^7x^7.$

82. $729a^{12} + 2916a^{10}c^2x^3 + 4860a^8c^4x^6 + 4320a^6c^6x^9$
$+ 2160a^4c^8x^{12} + 576a^2c^{10}x^{15} + 64c^{12}x^{18}.$

83. $b^5c^{10} - 10ab^4c^8y^3 + 40a^2b^3c^6y^6 - 80a^3b^2c^4a^4y^9 + 80a^4bc^2y^{12}$
$- 32a^5y^{15}.$

84. $256c^{16} - 768ac^{13}x^5 + 864a^2c^{10}x^{10} - 432a^3c^7x^{15} + 81a^4c^4x^{20}.$

85. $a^{10}b^{15} + 10a^{11}b^{13}x^4 + 40a^{12}b^{11}x^8 + 80a^{13}b^9x^{12} + 80a^{14}b^7x^{16}$
$+ 32a^{15}b^5x^{20}.$

86. $\frac{128a^7b^{14}}{2187} - \frac{112a^8b^{12}y}{243} + \frac{14a^9b^{10}y^2}{9} - \frac{35a^{10}b^8y^3}{12}$
$+ \frac{105a^{11}b^6y^4}{32} - \frac{567a^{12}b^4y^5}{256} + \frac{1701a^{13}b^2y^6}{2048} - \frac{2187a^{14}y^7}{16384}.$

87. $a^3\sqrt{a} + 7a^3x + 21a^2\sqrt{a} \times x^2 + 35a^2x^3 + 35a\sqrt{a}$
$\times x^4 + 21ax^5 + 7\sqrt{a} \times x^6 + x^7.$

88. $16b^4 - 64b^3\sqrt{2b} \times m + 224b^3m^2 - 224b^2\sqrt{2b}$
$\times m^3 + 280b^2m^4 - 112b\sqrt{2b} \times m^5 + 56bm^6 - 8\sqrt{2b}$
$\times m^7 + m^8.$

89. $27c^3\sqrt{3c} + 378c^3a + 756c^2\sqrt{3c} \times a^2 + 2520c^2a^3$
$+ 1680c\sqrt{3c} \times a^4 + 2016ca^5 + 448\sqrt{3c} \times a^6 + 128a^7.$

90. $\frac{a^4}{16} - 3a^3\sqrt{\tfrac{1}{2}a} \times y + \frac{63a^3y^2}{2} - 378a^2\sqrt{\tfrac{1}{2}a} \times y^3$
$+ \frac{2835a^2y^4}{2} - 6804a\sqrt{\tfrac{1}{2}a} \times y^5 + 10{,}206ay^6$
$- 17{,}496\sqrt{\tfrac{1}{2}a} \times y^7 + 6561y^8.$

91. $64e^3 + 192ae^2\sqrt{e} + 240a^2e^2 + 160a^3e\sqrt{e} + 60a^4e$
$+ 12a^5\sqrt{e} + a^6.$

92. $512b^9 - 2304b^8\sqrt{x} + 4608b^7x - 5376b^6x\sqrt{x}$
$+ 4032b^5x^2 - 2016b^4x^2\sqrt{x} + 672b^3x^3 - 144b^2x^3\sqrt{x}$
$+ 18bx^4 - x^4\sqrt{x}.$

93. $\frac{64a^6}{729} + \frac{64a^5\sqrt{2x}}{81} + \frac{160a^4x}{27} + \frac{320a^3x\sqrt{2x}}{27} + \frac{80a^2x^2}{3}$
$+ 16ax^2\sqrt{2x} + 8x^3.$

94. $\frac{243a^5}{1024} - \frac{405a^4\sqrt{\frac{1}{2}x}}{256} + \frac{135a^3x}{64} - \frac{45a^2x\sqrt{\frac{1}{2}x}}{16}$
$+ \frac{15ax^2}{16} - \frac{x^2\sqrt{\frac{1}{2}x}}{4}.$

95. $64a^6 - 576a^5\sqrt{y} + 2160a^4y - 4320a^3y\sqrt{y} + 4860a^2y^2$
$- 2916ay^2\sqrt{y} + 729y^3.$

96. $a^{14} + \frac{7a^{12}\sqrt{z}}{2} + \frac{21a^{10}z}{4} + \frac{35a^8z\sqrt{z}}{8} + \frac{35a^6z^2}{16}$
$+ \frac{21a^4z^2\sqrt{z}}{32} + \frac{7a^2z^3}{64} + \frac{z^3\sqrt{z}}{128}.$

97. $a^4\sqrt{a} + 9a^4\sqrt{x} + 36a^3x\sqrt{a} + 84a^3x\sqrt{x}$
$+ 126a^2x^2\sqrt{a} + 126a^2x^2\sqrt{x} + 84ax^3\sqrt{a}$
$+ 36ax^3\sqrt{x} + 9x^4\sqrt{a} + x^4\sqrt{x}.$

98. $b^4 - 8b^3\sqrt{by} + 28b^3y - 56b^2y\sqrt{by} + 70b^2y^2$
$- 56by^2\sqrt{by} + 28by^3 - 8y^3\sqrt{by} + y^4.$

99. $8c^3 + 24c^2\sqrt{6cx} + 180c^2x + 120cx\sqrt{6cx} + 270cx^2$
$+ 54x^2\sqrt{6cx} + 27x^3.$

100. $\frac{a^3\sqrt{\frac{1}{2}a}}{8} - \frac{7a^3\sqrt{2x}}{8} + \frac{21a^2x\sqrt{\frac{1}{2}a}}{2} - \frac{35a^2x\sqrt{2x}}{2}$
$+ 70ax^2\sqrt{\frac{1}{2}a} - 42ax^2\sqrt{2x} + 56x^3\sqrt{\frac{1}{2}a} - 8x^3\sqrt{2x}.$

101. $\frac{8a^3}{27}+\frac{8a^2\sqrt{\frac{1}{2}ax}}{3}+5a^2x+10ax\sqrt{\frac{1}{2}ax}+\frac{45ax^2}{8}$
$+\frac{27x^2\sqrt{\frac{1}{2}ax}}{8}+\frac{27x^3}{64}.$

102. $a^{15}-10a^{13}x^2\sqrt{ax}+45a^{12}x^5-120a^{10}x^7\sqrt{ax}$
$+210a^9x^{10}-252a^7x^{12}\sqrt{ax}+210a^6x^{15}-120a^4x^{17}\sqrt{ax}$
$+45a^3x^{20}-10ax^{22}\sqrt{ax}+x^{25}.$

103. $b^{22}\sqrt{b}+9b^{20}y^2+36b^{17}y^4\sqrt{b}+84b^{15}y^6+126b^{12}y^8\sqrt{b}$
$+126b^{10}y^{10}+84b^7y^{12}\sqrt{b}+36b^5y^{14}+9b^2y^{16}\sqrt{b}+y^{18}.$

104. $\frac{8h^9}{27}+\frac{8h^7x^2\sqrt{\frac{1}{3}h}}{3}+\frac{10h^6x^4}{3}+\frac{20h^4x^6\sqrt{\frac{1}{3}h}}{3}+\frac{5h^3x^8}{2}$
$+\frac{3hx^{10}\sqrt{\frac{1}{3}h}}{2}+\frac{x^{12}}{8}.$

105. $4a^2\sqrt[3]{4a^2}+32a^2\sqrt[3]{6ax^2}+112a^2x\sqrt[3]{9x}+336ax^2\sqrt[3]{4a^2}$
$+420ax^2\sqrt[3]{6ax^2}+336ax^3\sqrt[3]{9x}+252x^4\sqrt[3]{4a^2}$
$+72x^4\sqrt[3]{6ax^2}+9x^5\sqrt[3]{9x}.$

106. $32a^5x^5+80a^4x^4\sqrt[3]{3xy^2}+80a^3x^3y\sqrt[3]{9x^2y}+120a^2x^3y^2$
$+30ax^2y^2\sqrt[3]{3xy^2}+3xy^3\sqrt[3]{9x^2y}.$

107. $27a^3y^3\sqrt{3ay}+189a^3y^3\sqrt[3]{2x^2y}+189a^2xy^3\sqrt[6]{432a^3x^2y}$
$+630a^2x^2y^3+210ax^2y^3\sqrt[6]{108a^3x^4y^5}+126ax^3y^3\sqrt[3]{4xy^2}$
$+28x^4y^3\sqrt{3ay}+4x^4y^3\sqrt[3]{2x^2y}.$

108. $6561a^8b^8x^{24}-17{,}496a^7b^7x^{21}\sqrt[3]{5b^2xy^2}$
$+20{,}412a^6b^7x^{18}y\sqrt[3]{25bx^2y}-68{,}040a^5b^7x^{16}y^2$
$+28{,}350a^4b^6x^{13}y^2\sqrt[3]{5b^2xy^2}-7560a^3b^6x^{10}y^3\sqrt[3]{25bx^2y}$
$+6300a^2b^6x^8y^4-600ab^5x^5y^4\sqrt[3]{5b^2xy^2}$
$+25b^5x^2y^5\sqrt[3]{25bx^2y}.$

109. $\frac{x^5}{a^{10}}+\frac{10x^4\sqrt[6]{x^3y^2}}{a^9b}+\frac{45x^4\sqrt[3]{y^2}}{a^8b^2}+\frac{120x^3y\sqrt{x}}{a^7b^3}$
$+\frac{210x^3y\sqrt[3]{y}}{a^6b^4}+\frac{252x^2y\sqrt[6]{x^3y^4}}{a^5b^5}+\frac{210x^2y^2}{a^4b^6}$
$+\frac{120xy^2\sqrt[6]{x^3y^2}}{a^3b^7}+\frac{45xy^2\sqrt[3]{y^2}}{a^2b^8}+\frac{10y^3\sqrt{x}}{ab^9}+\frac{y^3\sqrt[3]{y}}{b^{10}}.$

110. $\frac{x^4\sqrt[3]{x^2}}{a^{14}}+\frac{7x^4\sqrt[4]{xy^3}}{a^{12}b}+\frac{21x^3y\sqrt[6]{x^5y^3}}{a^{10}b^2}+\frac{35x^3y^2\sqrt[12]{x^5y^3}}{a^8b^3}$
$+\frac{35x^3y^3}{a^6b^4}+\frac{21x^2y^3\sqrt[12]{x^7y^9}}{a^4b^5}+\frac{7x^2y^4\sqrt[6]{xy^3}}{a^2b^6}+\frac{xy^5\sqrt[4]{x^3y}}{b^7}.$

111. $\frac{x^3z^3}{y^6}-\frac{6z^2\sqrt[6]{a^2x^3y^4z^3}}{y^5}+\frac{15z^2\sqrt[3]{a^2y}}{x^2y^3}-\frac{20az\sqrt{xz}}{x^5y}$
$+\frac{15az\sqrt[3]{ay^2}}{x^7}-\frac{6ay^2\sqrt[6]{a^4x^3y^2z^3}}{x^{10}}+\frac{a^2y^4}{x^{12}}.$

112. $128a^7-448a^6x\sqrt{-1}-672a^5x^2+560a^4x^3\sqrt{-1}$
$+280a^3x^4-84a^2x^5\sqrt{-1}-14ax^6+x^7\sqrt{-1}.$

113. $729b^{12}+2916b^{10}x\sqrt{-1}-4860b^8x^2-4320b^6x^3\sqrt{-1}$
$+2160b^4x^4+576b^2x^5\sqrt{-1}-64x^6.$

114. $a^7\sqrt{a}+5a^6i\sqrt{x}-10a^4x\sqrt{a}-10a^3xi\sqrt{x}$
$+5ax^2\sqrt{a}+x^2i\sqrt{x}.$

115. $a^4+6a^3i\sqrt[12]{a^4x^9}-15a^2x\sqrt[6]{a^4x^3}-20a^2x^2i\sqrt[4]{x}$
$+15ax^3\sqrt[3]{a}+6x^3i\sqrt[12]{a^8x^9}-x^4\sqrt{x}.$

116. $a^6-9a^5i\sqrt[3]{a}-36a^4\sqrt[3]{a^2}+84a^4i+126a^3\sqrt[3]{a}$
$-126a^2i\sqrt[3]{a^2}-84a^2+36ai\sqrt[3]{a}+9\sqrt[3]{a^2}-i.$

117. $-b^5-10b^4\sqrt{b}-45b^4-120b^3\sqrt{b}-210b^3-252b^2\sqrt{b}$
$-210b^2-120b\sqrt{b}-45b-10\sqrt{b}-1.$

118. $(x^6 - 9x^5\sqrt[6]{8x^2} + 72x^4\sqrt[3]{x^2} - 168x^4\sqrt{2} + 504x^3\sqrt[3]{x}$
$- 504x^2\sqrt[6]{8x^4} + 672x^2 - 288x\sqrt[6]{8x^2} + 144\sqrt[3]{x^2}$
$- 16\sqrt{2})i.$

119. $-(y^5\sqrt[5]{y^3} + 7y^4\sqrt[15]{5^5y^{12}} + 21y^4\sqrt[3]{25} + 175y^3\sqrt[5]{y}$
$+ 175y^2\sqrt[15]{5^5y^6} + 105y\sqrt[15]{25^5y^9} + 175\sqrt[5]{y^4} + 25\sqrt[3]{5})i.$

120. $\frac{256b^{12}}{y^8} - \frac{1024b^9d\sqrt[6]{b^3y^4}}{y^7} + \frac{1792b^7d^2\sqrt[3]{y}}{y^5} - \frac{1792b^4d^3\sqrt{b}}{y^3}$
$+ \frac{1120b^2d^4\sqrt[3]{y^2}}{y^2} - \frac{448d^5\sqrt[6]{b^3y^2}}{b} + \frac{112d^6y^2}{b^3}$
$- \frac{16d^7y^3\sqrt[6]{b^3y^4}}{b^6} + \frac{d^8y^5\sqrt[3]{y}}{b^8}.$

121. $8{,}200{,}192a^6.$

122. $352{,}716a^{11}d^{10}.$

123. $326{,}592x^{16}.$

124. $-29{,}753{,}610{,}120a^{18}x^{55}.$

125. $119{,}759{,}850x^{34}.$

126. $-12{,}033{,}222{,}880y^{31}.$

127. $\frac{1547a^{11}x^9}{256}.$

128. $-165a^4x^2.$

129. $-1287a^4x^5i.$

130. $36{,}146{,}147{,}370{,}366{,}245{,}273{,}600b^{21}c^{58}.$

131. The coefficient of the nth term from the beginning is

$$\frac{m(m-1)(m-2)\ldots\ldots(m-n+2)}{1\times2\times3\ldots\ldots(n-1)}.$$

If the numerator and denominator are multiplied by $1\times2\times3\ldots\ldots(m-n+1)$, this expression becomes

$$\frac{\lfloor m}{\lfloor n-1\ \lfloor m-n+1}. \qquad (A)$$

The nth term from the end is the $(m+1)-(n-1)$, or the $m-n+2$ from the beginning, and its coefficient is

$$\frac{m(m-1)(m-2)\ldots\ldots[m-(m-n+2)+2]}{1\times2\times3\ldots\ldots m-n+1}$$
$$=\frac{m(m-1)(m-2)\ldots\ldots n}{1\times2\times3\ldots\ldots(m-n+1)}.$$

If these two terms are multiplied by $1\times2\times3\ldots\ldots(n-1)$, the second member of the last equality becomes

$$\frac{\underline{|m}}{\underline{|n-1}\,\underline{|m-n+1}},$$

an expression equal to (A).

132. The development of $(a+b)^m$ includes $m+1$ terms, of which the nth is

$$\frac{m(m-1)(m-2)\ldots\ldots(m-n+2)}{1\times2\times3\ldots\ldots(n-1)}a^{m-n+1}b^{n-1}.$$

The coefficient of the $(n+1)$ term will be formed by multiplying the preceding coefficient by $\frac{m-n+1}{n}$. This product will increase so much that we shall have $\frac{m-n+1}{n}>1$ or $n<\frac{m+1}{2}$. The maximum will have $\frac{m+1}{2}$ for n. As n is integral, if m is even, the term affected with the greatest coefficient will be marked by the integral number immediately superior to $\frac{m+1}{2}$. If m is uneven, there will be two terms whose coefficients will be maximum, namely, the $\left(\frac{m+1}{2}\right)$ term from the beginning and following it, the $\left(\frac{m+1}{2}\right)$ term from the end.

133. The greatest coefficient is:

1. Of $(a+b^3)^{21}$, the eleventh and twelfth $= 352,716$.
2. Of $(b+x^2)^{16}$, the ninth $= 12,870$.
3. Of $(1+y^5)^{32}$, the seventeenth $= 601,080,390$.

134. The coefficients of the development of $(1+a)^m$ are identically the same as those of the development of $(1-a)^m$. Then we have

$$(1-a)^m = 1 - \frac{m}{1}a + \frac{m(m-1)}{1\times 2}a^2 - \frac{m(m-1)(m-2)}{1\times 2\times 3}a^3 + \ldots\ldots$$

When m is entire and positive, the terms of uneven rank are all positive, and those of even rank negative. If $a=1$, the two members cancel each other. Then the sum of the terms of even rank equals that of the terms of uneven rank.

135. $$(1+x)^m = 1 + \frac{m}{1}x + \frac{m(m-1)}{1\times 2}x^2 + \frac{m(m-1)(m-2)}{1\times 2\times 3}x^3 + \ldots\ldots$$

or $$(1+x)^m - 1 = \frac{m}{1}x + \frac{m(m-1)}{1\times 2}x^2 + \frac{m(m-1)(m-2)}{1\times 2\times 3}x^3 + \ldots\ldots$$

Making $x=1$, we have

$$2^m - 1 = \frac{m}{1} + \frac{m(m-1)}{1\times 2} + \frac{m(m-1)(m-2)}{1\times 2\times 3} + \ldots\ldots$$

136. $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots\ldots$

(i.) $1 - \frac{1}{9} + \frac{1}{9^2} - \frac{1}{9^3} + \frac{1}{9^4} - \frac{1}{9^5} + \ldots\ldots = 0.9.$

(ii.) $1-\frac{7}{100}+\left(\frac{7}{100}\right)^2-\left(\frac{7}{100}\right)^3+\left(\frac{7}{100}\right)^4-\left(\frac{7}{100}\right)^5+\cdots\cdots$

$=\frac{3100}{3317}=0.93457943\ldots\ldots$

(iii.) $1+\frac{9}{1000}+\left(\frac{9}{1000}\right)^2+\left(\frac{9}{1000}\right)^3+\cdots\cdots=\frac{1000}{991}$

$=1.009081735\ldots\ldots$

137. $(1+x)^{\frac{1}{2}}=1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5x^4}{128}+\frac{7x^5}{256}-\cdots\cdots$

$=1+\frac{3}{16}-\frac{9}{512}+\frac{27}{8192}-\frac{405}{524108}+\frac{1701}{8388608}-\cdots\cdots$

$=1.1726.$

138. $(1+x)^{-\frac{1}{2}}=1-\frac{x}{2}+\frac{3x^2}{8}-\frac{5x^3}{16}+\frac{35x^4}{128}-\frac{63x^5}{256}+\cdots\cdots$

$=0.998503367\ldots\ldots$

139. $(1+x)^{\frac{1}{3}}=1+\frac{x}{3}-\frac{x^2}{9}+\frac{5x^3}{81}-\frac{10x^4}{243}+\frac{22x^5}{729}-\frac{154x^6}{6561}+\cdots\cdots$

(i.) 1.16553; (ii.) 0.96905; (iii.) 1.04004.

140. $(1+x)^{-\frac{1}{3}}=1-\frac{x}{3}+\frac{2x^2}{9}-\frac{14x^3}{81}+\frac{35x^4}{243}-\frac{91x^5}{729}$

$+\frac{728x^6}{6561}-\frac{1976x^7}{19683}+\cdots\cdots$

(i.) 1.069178.....; (iii.) 1.0455159.....

(ii.) 1.000667555.....;

141. $(1-x)^{\frac{2}{5}}=1-\frac{2x}{5}-\frac{3x^2}{25}-\frac{8x^3}{125}-\frac{26x^4}{625}-\frac{468x^5}{15625}$

$-\frac{1794x^6}{78125}-\cdots\cdots=0.998398075\ldots\ldots$ when $x=0.004$.

142. $(1+15)^{\frac{1}{4}}=15^{\frac{1}{4}}\left(1+\frac{1}{4\times 15}-\frac{3}{32\times 15^2}+\frac{7}{128\times 15^3}\right.$

$\left.-\frac{77}{2048\times 15^4}+\frac{231}{8192\times 15^5}-\cdots\cdots\right).$

143. $$27^{\frac{1}{3}}\left(1+\frac{2}{27}\right)^{\frac{1}{3}}=27^{\frac{1}{3}}\left(1+\frac{2}{3\times27}-\frac{2^2}{3^2\times27^2}+\frac{5\times2^3}{3^4\times27^3}-\frac{10\times2^4}{3^5\times27^4}+\frac{22\times2^5}{3^6\times27^5}-\frac{154\times2^6}{3^8\times27^6}+\dots\right).$$

144. $$9^{-2}\left(1+\frac{1}{9}\right)^{-2}=\frac{1}{81}\left(1-\frac{2}{9}+\frac{1}{27}-\frac{4}{9^3}+\frac{5}{9^4}-\frac{6}{9^5}+\frac{7}{9^6}-\frac{8}{9^7}+\dots\right).$$

145. $$64^{\frac{1}{2}}\left(1+\frac{1}{64}\right)^{\frac{1}{2}}=8\left(1+\frac{1}{2\times64}-\frac{1}{2^3\times64^2}+\frac{1}{2^4\times64^3}-\frac{5}{2^7\times64^4}+\frac{7}{2^8\times64^5}-\frac{21}{2^{10}\times64^6}+\dots\right).$$

146. $$a^{-5}-5a^{-6}x+15a^{-7}x^2-35a^{-8}x^3+70a^{-9}x^4-126a^{-10}x^5+210a^{-11}x^6-330a^{-12}x^7+\dots$$

147. $$a^{-6}+6a^{-7}x+21a^{-8}x^9+56a^{-9}x^3+126a^{-10}x^4+252a^{-11}x^5+462a^{-12}x^6+792a^{-13}x^7+\dots$$

148. $$\frac{1}{64b^6}+\frac{3y}{64b^7}+\frac{21y^2}{256b^8}+\frac{7y^3}{64b^9}+\frac{63y^4}{512b^{10}}+\frac{63y^5}{512b^{11}}+\frac{231y^6}{2048b^{12}}+\frac{99y^7}{1024b^{13}}+\dots$$

149. $$\frac{512}{c^9}-\frac{9216z}{c^{10}}+\frac{92160z^2}{c^{11}}-\frac{675840z^3}{c^{12}}+\frac{4055040z^4}{c^{13}}-\frac{21086208z^5}{c^{14}}+\frac{98402304z^6}{c^{15}}-\frac{421724160z^7}{c^{16}}+\dots$$

150. $$\frac{1}{6561}+\frac{8x}{19683}+\frac{4x^2}{6561}+\frac{40x^3}{59049}+\frac{110x^4}{177147}+\frac{88x^5}{177147}+\frac{572x^6}{1594323}+\frac{1144x^7}{4782969}+\dots$$

151. $\frac{1}{a^9}-\frac{9x}{2a^{10}}+\frac{45x^2}{4a^{11}}-\frac{165x^3}{8a^{12}}+\frac{495x^4}{16a^{13}}-\frac{1287x^5}{32a^{14}}+\frac{3003x^6}{64a^{15}}$
$-\frac{6435x^7}{128a^{16}}+\cdots\cdots$

152. $1-10x+55x^2-220x^3+715x^4-2002x^5+5005x^6$
$-11{,}440x^7+\cdots\cdots$

153. $1+11y+66y^2+286y^3+1001y^4+3003y^5+8008y^6$
$+19{,}448y^7+\cdots\cdots$

154. $\frac{1}{z^9}-\frac{9}{z^{10}}+\frac{45}{z^{11}}-\frac{165}{z^{12}}+\frac{495}{z^{13}}-\frac{1287}{z^{14}}+\frac{3003}{z^{15}}-\frac{6435}{z^{16}}+\cdots\cdots$

155. $1+x+x^2+x^3+x^4+x^5+x^6+x^7+\cdots\cdots$

156. $\frac{1}{a^{10}}-\frac{5x^3}{a^{12}}+\frac{15x^6}{a^{14}}-\frac{35x^9}{a^{16}}+\frac{70x^{12}}{a^{18}}-\frac{126x^{15}}{a^{20}}+\frac{210x^{18}}{a^{22}}$
$-\frac{330x^{21}}{a^{24}}+\cdots\cdots$

157. $\frac{1}{b^{24}}+\frac{8x^2}{b^{27}}+\frac{36x^4}{b^{30}}+\frac{120x^6}{b^{33}}+\frac{330x^8}{b^{36}}+\frac{792x^{10}}{b^{39}}+\frac{1716x^{12}}{b^{42}}$
$+\frac{3432x^{14}}{b^{45}}+\cdots\cdots$

158. $\frac{1}{c^9}+\frac{18x^3}{c^{10}}+\frac{180x^6}{c^{11}}+\frac{1320x^9}{c^{12}}+\frac{7920x^{12}}{c^{13}}+\frac{41184x^{15}}{c^{14}}$
$+\frac{192192x^{18}}{c^{15}}+\frac{823680x^{21}}{c^{16}}+\cdots\cdots$

159. $\frac{1}{a^{12}c^6}-\frac{6x^2}{a^{13}c^7}+\frac{21x^4}{a^{14}c^8}-\frac{56x^6}{a^{15}c^9}+\frac{126x^8}{a^{16}c^{10}}-\frac{252x^{10}}{a^{17}c^{11}}+\frac{462x^{12}}{a^{18}c^{12}}$
$-\frac{792x^{14}}{a^{19}c^{13}}+\cdots\cdots$

160. $\frac{1}{a^3}+\frac{6x^2}{a^3\sqrt{a}}+\frac{21x^4}{a^4}+\frac{56x^6}{a^4\sqrt{a}}+\frac{126x^8}{a^5}+\frac{252x^{10}}{a^5\sqrt{a}}$
$+\frac{462x^{12}}{a^6}+\frac{792x^{14}}{a^6\sqrt{a}}+\dots\dots$

161. $\sqrt{b}+\frac{h}{2\sqrt{b}}-\frac{h^2}{8b\sqrt{b}}+\frac{h^3}{16b^2\sqrt{b}}-\frac{5h^4}{128b^3\sqrt{b}}$
$+\frac{7h^5}{256b^4\sqrt{b}}-\frac{21h^6}{1024b^5\sqrt{b}}+\frac{33h^7}{2048b^6\sqrt{b}}-\dots\dots$

162. $\sqrt[4]{b}-\frac{x}{4\sqrt[4]{b^3}}-\frac{3x^2}{32b\sqrt[4]{b^3}}-\frac{7x^3}{128b^2\sqrt[4]{b^3}}-\frac{77x^4}{2048b^3\sqrt[4]{b^3}}$
$-\frac{231x^5}{8192b^4\sqrt[4]{b^3}}-\frac{1463x^6}{65536b^5\sqrt[4]{b^3}}-\frac{4807x^7}{262144b^6\sqrt[4]{b^3}}-\dots\dots$

163. $\sqrt[5]{2a}+\frac{y}{5\sqrt[5]{16a^4}}-\frac{y^2}{25a\sqrt[5]{16a^4}}+\frac{3y^3}{250a^2\sqrt[5]{16a^4}}$
$-\frac{21y^4}{5000a^3\sqrt[5]{16a^4}}+\frac{399y^5}{250000a^4\sqrt[5]{16a^4}}$
$-\frac{399y^6}{625000a^5\sqrt[5]{16a^4}}+\frac{1653y^7}{6250000a^6\sqrt[5]{16a^4}}-\dots\dots$

164. $\sqrt[6]{2}+\frac{a}{6\sqrt[6]{2^5}}-\frac{5a^2}{144\sqrt[6]{2^5}}+\frac{55a^3}{5184\sqrt[6]{2^5}}-\frac{935a^4}{248832\sqrt[6]{2^5}}$
$+\frac{4301a^5}{2985984\sqrt[6]{2^5}}-\frac{124729a^6}{214990848\sqrt[6]{2^6}}$
$+\frac{623645a^7}{2579890176\sqrt[6]{2^5}}-\dots\dots$

165. $x+\frac{a}{2x}-\frac{a^2}{8x^3}+\frac{a^3}{16x^5}-\frac{5a^4}{128x^7}+\frac{7a^5}{256x^9}-\frac{21a^6}{1024x^{11}}$
$+\frac{33a^7}{2048x^{13}}-\dots\dots$

166. $$a-\frac{1}{2a}-\frac{1}{8a^3}-\frac{1}{16a^5}-\frac{5}{128a^7}-\frac{7}{256a^9}-\frac{21}{1024a^{11}}-\frac{33}{2048a^{13}}-\dots\dots$$

167. $$\sqrt[7]{a}+\frac{2b}{7\sqrt[7]{a^6}}-\frac{12b^2}{49a\sqrt[7]{a^6}}+\frac{104b^3}{343a^2\sqrt[7]{a^6}}-\frac{1040b^4}{2401a^3\sqrt[7]{a^6}}+\frac{11232b^5}{16807a^4\sqrt[7]{a^6}}-\frac{127296b^6}{117649a^5\sqrt[7]{a^6}}+\frac{10438272b^7}{5764801a^6\sqrt[7]{a^6}}-\dots\dots$$

168. $$1+\frac{a}{9}-\frac{4a^2}{81}+\frac{68a^3}{2187}-\frac{442a^4}{19683}+\frac{3094a^5}{177147}-\frac{68068a^6}{4782969}+\frac{515372a^7}{43046721}-\dots\dots$$

169. $$1+\frac{y}{10}-\frac{9y^2}{200}+\frac{57y^3}{2000}-\frac{1653y^4}{80000}+\frac{64467y^5}{4000000}-\frac{1052961y^6}{80000000}+\frac{8874957y^7}{800000000}-\dots\dots$$

170. $$1+\frac{z^2}{9}-\frac{4z^4}{81}+\frac{68z^6}{2187}-\frac{442z^8}{19683}+\frac{3094z^{10}}{177147}-\frac{68068z^{12}}{4782969}+\frac{515372z^{14}}{43046721}-\dots\dots$$

171. $$1+\frac{3h}{4}-\frac{3h^2}{32}+\frac{5h^3}{128}-\frac{45h^4}{2048}+\frac{117h^5}{8192}-\frac{663h^6}{65536}+\frac{1989h^7}{262144}-\dots\dots$$

172. $$\sqrt[7]{2^4}-\frac{4y^3}{7\sqrt[7]{2^3}}-\frac{3y^6}{49\sqrt[7]{2^3}}-\frac{5y^9}{343\sqrt[7]{2^3}}-\frac{85y^{12}}{19208\sqrt[7]{2^3}}-\frac{51y^{15}}{33614\sqrt[7]{2^3}}-\frac{527y^{18}}{941192\sqrt[7]{2^3}}-\frac{10013y^{21}}{46118408\sqrt[7]{2^3}}-\dots\dots$$

173. $\sqrt[5]{3^3}+\frac{3d^4}{5\sqrt[5]{3^2}}-\frac{d^8}{25\sqrt[5]{3^2}}+\frac{7d^{12}}{1125\sqrt[5]{3^2}}-\frac{7d^{16}}{5625\sqrt[5]{3^2}}$
$+\frac{119d^{20}}{421875\sqrt[5]{3^2}}-\frac{1309d^{24}}{18984375\sqrt[5]{3^2}}$
$+\frac{187d^{28}}{10546875\sqrt[5]{3^2}}-\dots\dots$

174. $a^2\sqrt[5]{a^2}-\frac{4b}{5\sqrt[5]{a^3}}-\frac{2b^2}{25a^3\sqrt[5]{a^3}}-\frac{4b^3}{125a^6\sqrt[5]{a^3}}-\frac{11b^4}{625a^9\sqrt[5]{a^3}}$
$-\frac{176b^5}{15625a^{12}\sqrt[5]{a^3}}-\frac{616b^6}{78125a^{15}\sqrt[5]{a^3}}$
$-\frac{2288b^7}{390625a^{18}\sqrt[5]{a^3}}-\dots\dots$

175. $y\sqrt[4]{y^3}-\frac{7b^3}{8\sqrt[4]{y}}-\frac{7b^6}{128y^2\sqrt[4]{y}}-\frac{21b^9}{1024y^4\sqrt[4]{y}}$
$-\frac{357b^{12}}{32768y^6\sqrt[4]{y}}-\frac{1785b^{15}}{262144y^8\sqrt[4]{y}}-\frac{19635b^{18}}{4194304y^{10}\sqrt[4]{y}}$
$-\frac{115005b^{21}}{33554432y^{12}\sqrt[4]{y}}-\dots\dots$

176. $\frac{1}{a\sqrt{a}}-\frac{1}{2a^4\sqrt{a}}+\frac{3}{8a^7\sqrt{a}}-\frac{5}{16a^{10}\sqrt{a}}+\frac{35}{128a^{13}\sqrt{a}}$
$-\frac{63}{256a^{16}\sqrt{a}}+\frac{231}{1024a^{19}\sqrt{a}}-\frac{429}{2048a^{22}\sqrt{a}}+\dots\dots$

177. $\frac{1}{x}+\frac{a}{2x^3}+\frac{3a^2}{8x^5}+\frac{5a^3}{16x^7}+\frac{35a^4}{128x^9}+\frac{63a^5}{256x^{11}}+\frac{231a^6}{1024x^{13}}$
$+\frac{429a^7}{2048x^{15}}+\dots\dots$

178. $1+\frac{x^5}{3}+\frac{2x^{10}}{9}+\frac{14x^{15}}{81}+\frac{35x^{20}}{243}+\frac{91x^{25}}{729}+\frac{728x^{30}}{6561}+\frac{1976x^{35}}{19683}+\ldots\ldots$

179. $1-\frac{2d}{5}+\frac{12d^2}{25}-\frac{88d^3}{125}+\frac{704d^4}{625}-\frac{29568d^5}{15625}+\frac{256256d^6}{78125}-\frac{2269696d^7}{390625}+\ldots\ldots$

180. $\frac{1}{8}-\frac{3h}{256}+\frac{3h^2}{2048}-\frac{13h^3}{65536}+\frac{117h^4}{4194304}-\frac{2691h^5}{671088640}+\frac{6279h^6}{10737418240}-\frac{29601h^7}{343597383680}+\ldots\ldots$

181. $\frac{1}{27}+\frac{x^2}{81}+\frac{5x^4}{1458}+\frac{35x^6}{39366}+\frac{35x^8}{157464}+\frac{77x^{10}}{1417176}+\frac{1001x^{12}}{76527504}+\frac{715x^{14}}{229582512}+\ldots\ldots$

182. $\frac{1}{a^5}-\frac{15y^4}{7a^{12}}+\frac{270y^8}{49a^{19}}-\frac{5130y^{12}}{343a^{26}}+\frac{100035y^{16}}{2401a^{23}}-\frac{1980693y^{20}}{16807a^{40}}+\frac{39613860y^{24}}{117649a^{47}}-\frac{5585554260y^{28}}{5764801a^{54}}+\ldots\ldots$

183. $\sqrt[4]{a}-\frac{\sqrt[12]{a^9x^4}}{2b}+\frac{3a\sqrt[12]{a^3x^8}}{8b^2}-\frac{5ax\sqrt[4]{a^3}}{16b^3}+\frac{35a^2x\sqrt[12]{a^3x^4}}{128b^4}-\frac{63a^2x\sqrt[12]{a^9x^8}}{256b^5}+\frac{231a^3x^2\sqrt[4]{a}}{1024b^6}-\frac{429a^3x^2\sqrt[12]{a^9x^4}}{2048b^7}+\ldots\ldots$

184. $\frac{\sqrt[3]{x}}{\sqrt[9]{a^2}}-\frac{x^3\sqrt[3]{x}}{3a\sqrt[9]{a^8}}+\frac{2x^6\sqrt[3]{x}}{9a^3\sqrt[9]{a^5}}-\frac{14x^9\sqrt[3]{x}}{81a^5\sqrt[9]{a^2}}+\frac{35x^{12}\sqrt[3]{x}}{243a^6\sqrt[9]{a^8}}-\frac{91x^{15}\sqrt[3]{x}}{729a^8\sqrt[9]{a^5}}+\frac{728x^{18}\sqrt[3]{x}}{6561a^{10}\sqrt[9]{a^2}}-\frac{1976x^{21}\sqrt[3]{x}}{19683a^{11}\sqrt[9]{a^8}}+\ldots\ldots$

185. $2a\sqrt{2a}+\frac{24a^2\sqrt{2a}}{x^2}+\frac{240a^3\sqrt{2a}}{x^4}+\frac{2240a^4\sqrt{2a}}{x^6}$
$+\frac{20160a^5\sqrt{2a}}{x^8}+\frac{177408a^6\sqrt{2a}}{x^{10}}+\frac{1537536a^7\sqrt{2a}}{x^{12}}$
$+\frac{13178880a^8\sqrt{2a}}{x^{14}}+\ldots\ldots$

186. $\frac{1}{\sqrt[4]{8}}-\frac{3}{8}\sqrt[4]{\frac{x^3}{4}}+\frac{21x}{128}\sqrt[4]{\frac{x^2}{2}}-\frac{77x^2\sqrt[4]{x}}{1024}+\frac{1155x^3}{16384\sqrt[4]{8}}$
$-\frac{4389x^3}{131072}\sqrt[4]{\frac{x^3}{4}}+\frac{33649x^4}{2097152}\sqrt[4]{\frac{x^2}{2}}-\frac{129789x^5\sqrt[4]{x}}{16777216}+\ldots\ldots$

187. $\frac{1}{a^6}-\frac{3x^3}{a^8}+\frac{6x^6}{a^{10}}-\frac{10x^9}{a^{12}}+\frac{15x^{12}}{a^{14}}-\frac{21x^{15}}{a^{16}}+\frac{28x^{18}}{a^{18}}$
$-\frac{36x^{21}}{a^{20}}+\ldots\ldots$

188. $\frac{1}{a^2}+\frac{2x^3}{3a^5}+\frac{5x^6}{9a^8}+\frac{40x^9}{81a^{11}}+\frac{110x^{12}}{243a^{14}}+\frac{308x^{15}}{729a^{17}}+\frac{2618x^{18}}{6561a^{20}}$
$+\frac{7480x^{21}}{19683a^{23}}+\ldots\ldots$

189. $a^3-7a^3\sqrt[3]{b}+21a^2\sqrt[6]{a^3b^4}-35a^2b+35ab\sqrt[6]{a^3b^2}$
$-21ab\sqrt[3]{b^2}+7b^2\sqrt{a}-b^2\sqrt[3]{b}.$

190. $\frac{1}{a}-\frac{p}{a^2}+\frac{p^2}{a^3}-\frac{p^3}{a^4}+\frac{p^4}{a^5}-\frac{p^5}{a^6}+\frac{p^6}{a^7}-\frac{p^7}{a^8}+\cdots$

191. $\frac{1}{b^6}-\frac{3r}{b^8}+\frac{6r^2}{b^{10}}-\frac{10r^3}{b^{12}}+\frac{15r^4}{b^{14}}-\frac{21r^5}{b^{16}}+\frac{28r^6}{b^{18}}-\frac{36r^7}{b^{20}}+\ldots\ldots$

192. $$-\left(\frac{1}{H^5}+\frac{5t}{H^6}+\frac{15t^2}{H^7}+\frac{35t^3}{H^8}+\frac{70t^4}{H^9}+\frac{126t^5}{H^{10}}+\frac{210t^6}{H^{11}}+\frac{330t^7}{H^{12}}+\ldots\ldots\right).$$

193. $$\frac{1}{Y^4}+\frac{4v}{Y^5}+\frac{10v^2}{Y^6}+\frac{20v^3}{Y^7}+\frac{35v^4}{Y^8}+\frac{56v^5}{Y^9}+\frac{84v^6}{Y^{10}}+\frac{120v^7}{Y^{11}}+\ldots\ldots$$

194. $$-\left(\frac{1}{\sqrt[3]{4a^2}}+\frac{1}{4a^2\sqrt[3]{4a^2}}+\frac{1}{8a^4\sqrt[3]{4a^2}}+\frac{7}{96a^6\sqrt[3]{4a^2}}+\frac{35}{768a^8\sqrt[3]{4a^2}}+\frac{91}{3072a^{10}\sqrt[3]{4a^2}}+\frac{91}{4608a^{12}\sqrt[3]{4a^2}}+\frac{247}{18432a^{14}\sqrt[3]{4a^2}}+\ldots\ldots\right).$$

195. $$\frac{1}{\sqrt[7]{x^5}}-\frac{2}{7x^5\sqrt[7]{x^5}}+\frac{16}{49x^{10}\sqrt[7]{x^5}}-\frac{160}{343x^{15}\sqrt[7]{x^5}}+\frac{1760}{2401x^{20}\sqrt[7]{x^5}}-\frac{20416}{16807x^{25}\sqrt[7]{x^5}}+\frac{244992}{117649x^{30}\sqrt[7]{x^5}}-\frac{21069312}{5764801x^{35}\sqrt[7]{x^5}}+\ldots\ldots$$

196. $$\frac{1}{\sqrt[15]{81^3x^8}}+\frac{8\sqrt[4]{a}}{15x\sqrt[5]{81x}}+\frac{8\sqrt{a}}{25x\sqrt[15]{81^3x^{13}}}+\frac{224\sqrt[4]{a^3}}{1125x^2\sqrt[15]{81^3x^8}}+\frac{2128a}{16875x^3\sqrt[5]{81x}}+\frac{34048a\sqrt[4]{a}}{421875x^3\sqrt[15]{81^3x^{13}}}+\frac{987392a\sqrt{a}}{18984375x^4\sqrt[15]{81^3x^8}}+\frac{9591808a\sqrt[4]{a^3}}{284765625x^5\sqrt[5]{81x}}+\ldots\ldots$$

197. $$\frac{1}{\sqrt[5]{a^4y^2}}-\frac{2\sqrt[3]{x}}{5ay\sqrt[5]{a^4y^2}}+\frac{7\sqrt[3]{x^2}}{25a^2y^2\sqrt[5]{a^4y^2}}-\frac{28x}{125a^3y^3\sqrt[5]{a^4y^2}}$$
$$+\frac{119x\sqrt[3]{x}}{625a^4y^4\sqrt[5]{a^4y^2}}-\frac{2618x\sqrt[3]{x^2}}{15625a^5y^5\sqrt[5]{a^4y^2}}$$
$$+\frac{11781x^2}{78125a^6y^6\sqrt[5]{a^4y^2}}-\frac{53856x^2\sqrt[3]{x}}{390625a^7y^7\sqrt[5]{a^4y^2}}+\ldots\ldots$$

198. $$z^3\sqrt[5]{81z}+\frac{4\sqrt[5]{h}}{5\sqrt[5]{3z^4}}-\frac{2\sqrt[5]{h^2}}{75z^4\sqrt[5]{3z^4}}+\frac{4\sqrt[5]{h^3}}{1125z^8\sqrt[5]{3z^4}}$$
$$-\frac{11\sqrt[5]{h^4}}{16875z^{12}\sqrt[5]{3z^4}}+\frac{176h}{1265625z^{16}\sqrt[5]{3z^4}}$$
$$-\frac{616h\sqrt[5]{h}}{18984375z^{20}\sqrt[5]{3z^4}}+\frac{2288h\sqrt[5]{h^2}}{284765625z^{24}\sqrt[5]{3z^4}}-\ldots\ldots$$

199. 3.162277 .....
200. 7.280109 .....
201. 4.431047 .....
202. 3.107232 .....
203. 2.871621 .....
204. 3.038504 .....
205. 4.073062 .....
206. 5.999889 .....
207. 0.309838 .....
208. 0.954640 .....
209. 5.873670 .....
210. 4.330127 .....
211. 3.878277 .....
212. 2.768621 .....
213. 5.028970 .....
214. 6.003033 .....
215. 2.021744 .....
216. 18.566355 .....
217. 4.998999 .....
218. 3.000457 .....
219. 1.028901 .....
220. 2.000020 .....
221. 6.846323 .....
222. 4.083680 .....

Ex. 58. Page 201.

1. $\frac{2}{5}$; $\frac{3}{5}$.

2. The number of cases favorable to the event being represented by m, and the number of cases unfavorable by n; then $m:n$ are the odds in favor of the event, and the probability of its happening is represented by the fraction $\frac{m}{m+n}$. (i.) If m is 0, the chance of drawing a white ball is $\frac{0}{10}$, or zero; that is, the bag contained no white ball. (ii.) If n is equal to 0, the bag contained 10 white and no black; therefore, the chance of drawing a white ball is $\frac{10}{10}$, or unity; that is, certainty. (iii.) If $n = m$, P becomes $\frac{m}{2m} = \frac{1}{2}$; and one of the five white balls is as likely to be drawn as one of the black ones.

3. $5\begin{cases}\text{4 black,}\\ \text{1 white.}\end{cases}$

4. $\frac{1}{5}$.

5. $\frac{1}{6}$; $\frac{1}{6}$; $\frac{5}{12}$.

6. 17 : 19.

7. 125 : 91 against.

8. 116 : 117.

9. $\frac{2}{11}$, $\frac{5}{33}$, $\frac{19}{33}$.

10. $\frac{1}{22}$.

11. $\frac{3}{11}$.

12. $\frac{1}{2}$; $\frac{3}{14}$; $\frac{1}{5}$; $\frac{3}{35}$; $\frac{29}{70}$; $\frac{32}{35}$.

13. 17 : 2 against.

14. The number of ways in which they could come 1, 3, 5, ..., is

$$8+\frac{8\times7\times6}{1\times2\times3}+\frac{8\times7\times6\times5\times4}{1\times2\times3\times4\times5}$$
$$+\frac{8\times7\times6\times5\times4\times3\times2\times1}{1\times2\times3\times4\times5\times6\times7}=128.$$

The number of ways in which they would come 2, 4, 6, ..., is

$$\frac{8\times 7}{1\times 2}+\frac{8\times 7\times 6\times 5}{1\times 2\times 3\times 4}+\frac{8\times 7\times 6\times 5\times 4\times 3}{1\times 2\times 3\times 4\times 5\times 6}=126.$$

Therefore, the ratio is 128 : 126 = 64 : 63. Similarly, for 5.

15. $\frac{1}{35}$. 16. $\frac{100}{357}$; $\frac{19}{714}$. 17. $\frac{1}{165}$; $\frac{27}{1331}$. 18. $\frac{14}{55}$; $\frac{336}{1331}$.

19. 11 : 5 20. Chance for each $= \frac{216}{625}$.

21. Because the second and third terms of $(\frac{1}{2}+\frac{1}{2})^5$ are alike.

22. $\frac{125}{216}$; $\frac{1}{216}$; $\frac{75}{216}$. 23. $\frac{17}{81}$. 24. $7\frac{5}{7}$ pence.

25. The general total of the chances is 216, and either 10 or 11 may be thrown in 27 different ways. The odds then are 27 : 189 or 1 : 7.

26. $\frac{3}{20}$; $\frac{1}{10}$.

Ex. 59. Page 204.

1. $4+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{2}}}.$

3. $7+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}.$

5. $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{33}}}.$

2. $3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3}}}.$

4. $20+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{4}}}.$

6. $\cfrac{1}{6+\cfrac{1}{4+\cfrac{1}{8+\cfrac{1}{3}}}}.$

7. $\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{21}}}}.$

8. $7+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}}.$

9. $$1+\cfrac{1}{16+\cfrac{1}{1+\cfrac{1}{12+\cfrac{1}{2}}}}.$$

10. $$3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{14}}}}}.$$

11. $$\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{2}}}}}}.$$

12. $$\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5}}}}}}.$$

13. $$9+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{61+\cfrac{1}{1+\cfrac{1}{3}}}}}}.$$

14. $$5+\cfrac{1}{14+\cfrac{1}{3+\cfrac{1}{2}}}.$$

15. $$\cfrac{1}{41+\cfrac{1}{2+\cfrac{1}{39+\cfrac{1}{1+\cfrac{1}{2}}}}}.$$

16. $$\cfrac{1}{a+\cfrac{1}{3a+\cfrac{1}{5a}}}.$$

17. $$\cfrac{1}{5m+\cfrac{1}{7m^2+\cfrac{1}{9m^3}}}.$$

18. $$\cfrac{1}{1+\cfrac{1}{x+\cfrac{1}{1+\cfrac{1}{x^2}}}}.$$

19. $$\cfrac{1}{x+\cfrac{1}{2x+\cfrac{1}{3x+\cfrac{1}{4x}}}}.$$

20. $x+\cfrac{1}{x^2+\cfrac{1}{x^3+\cfrac{1}{x^4}}}$.

21. $\cfrac{1}{a^3x+\cfrac{1}{a^2x^2+\cfrac{1}{ax^3+\cfrac{1}{x^4}}}}$.

22. $\frac{22}{51}$.

23. $\frac{9}{56}$.

24. $\frac{22}{179}$.

25. $\frac{5}{8}$.

26. $2\frac{36}{115}$.

27. $\frac{249}{56}$.

28. $3\frac{33}{74}$.

29. $7\frac{21}{130}$.

30. $1\frac{11}{25}$.

31. $5\frac{73}{99}$.

32. $\frac{3655}{47844}$.

33. $\frac{9307}{47844}$.

34. $\frac{1}{2}$; $\frac{3}{7}$; $\frac{13}{30}$; $\frac{68}{157}$; $\frac{421}{972}$.

35. $\frac{1}{3}$; $\frac{1}{4}$; $\frac{5}{19}$; $\frac{6}{23}$; $\frac{35}{134}$.

36. $\frac{1}{2}$; $\frac{7}{15}$; $\frac{22}{47}$; $\frac{205}{438}$; $\frac{842}{1799}$.

37. $\frac{1}{4}$; $\frac{8}{33}$; $\frac{129}{532}$; $\frac{266}{1097}$; $\frac{1193}{4920}$.

38. $\frac{1}{1}$; $\frac{3}{4}$; $\frac{16}{21}$; $\frac{115}{151}$; $\frac{1051}{1380}$; $\frac{11676}{15331}$.

39. $\frac{1}{2}$; $\frac{2}{5}$; $\frac{5}{12}$; $\frac{12}{29}$; $\frac{29}{70}$; $\frac{99}{239}$.

40. $\frac{3}{1}$; $\frac{4}{1}$; $\frac{23}{6}$; $\frac{27}{7}$; $\frac{941}{244}$; $\frac{968}{251}$.

41. $\frac{1}{4}$; $\frac{1}{5}$; $\frac{3}{14}$; $\frac{4}{19}$; $\frac{51}{242}$; $\frac{463}{2197}$.

42. $\frac{18}{1}$; $\frac{19}{1}$; $\frac{37}{2}$; $\frac{93}{5}$; $\frac{502}{27}$; $\frac{595}{32}$; $\frac{2882}{155}$; $\frac{6359}{342}$.

43. $\frac{1}{1}$; $\frac{4}{3}$; $\frac{5}{4}$; $\frac{9}{7}$; $\frac{167}{130}$; $\frac{176}{137}$; $\frac{343}{267}$; $\frac{862}{671}$.

44. $\frac{1}{1}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{7}$; $\frac{8}{11}$; $\frac{13}{18}$; $\frac{21}{29}$; $\frac{34}{47}$; $\frac{463}{640}$; $\frac{1423}{1967}$; $\frac{6155}{8508}$.

45. $\frac{1}{1}$; $\frac{5}{6}$; $\frac{11}{13}$; $\frac{16}{19}$; $\frac{187}{222}$; $\frac{577}{685}$; $\frac{2495}{2962}$; $\frac{10557}{12533}$; $\frac{13052}{15495}$.

46. $\frac{1}{1}$; $\frac{5}{4}$; $\frac{6}{5}$; $\frac{11}{9}$; $\frac{28}{23}$; $\frac{67}{55}$; $\frac{296}{243}$; $\frac{363}{298}$; $\frac{659}{541}$; $\frac{1022}{839}$.

47. $\frac{1}{2}$; $\frac{6}{13}$; $\frac{817}{1770}$; $\frac{2457}{5323}$; $\frac{3274}{7093}$; $\frac{5731}{12416}$; $\frac{20467}{44341}$.

48. $\frac{1}{11}$; $\frac{1}{12}$; $\frac{3}{35}$; $\frac{7}{82}$; $\frac{10}{117}$; $\frac{57}{667}$; $\frac{181}{2118}$; $\frac{962}{11257}$; $\frac{1143}{13375}$; $\frac{5534}{64757}$; etc.

49. $\frac{1}{1}$; $\frac{17}{16}$; $\frac{86}{81}$; $\frac{103}{97}$; $\frac{395}{372}$; $\frac{498}{469}$; $\frac{1889}{1779}$; $\frac{4276}{4027}$; $\frac{14717}{13860}$.

50. $\frac{3}{1}$; $\frac{22}{7}$; $\frac{333}{106}$; $\frac{355}{113}$; $\frac{86598}{27565}$; $\frac{86953}{27678}$; $\frac{173551}{55243}$.

51. $\frac{2}{1}$; $\frac{3}{1}$; $\frac{8}{3}$; $\frac{11}{4}$; $\frac{19}{7}$; $\frac{87}{32}$; $\frac{106}{39}$; $\frac{193}{71}$; $\frac{1264}{465}$; $\frac{1457}{536}$; etc.

52. $\frac{1}{2}$; $\frac{3}{7}$; $\frac{10}{23}$; $\frac{33}{76}$; $\frac{43}{99}$; $\frac{76}{175}$; $\frac{271}{624}$; $\frac{1702}{3919}$; $\frac{1973}{4543}$; etc.

53. $\frac{1}{1}$; $\frac{4}{3}$; $\frac{17}{13}$; $\frac{21}{16}$; $\frac{206}{157}$; $\frac{227}{173}$; $\frac{1795}{1368}$.

54. $3+\cfrac{1}{3+\cfrac{1}{6+\cdots}}$

55. $3+\cfrac{1}{1+\cfrac{1}{6+\cdots}}$

56. $4+\cfrac{1}{8+\cfrac{1}{8+\cfrac{1}{8+\cdots}}}$

57. $5+\cfrac{1}{5+\cfrac{1}{10+\cdots}}$

58. $6+\cfrac{1}{12+\cfrac{1}{12+\cfrac{1}{12+\cdots}}}$

59. $7+\cfrac{1}{7+\cfrac{1}{14+\cdots}}$

60. $6+\cfrac{1}{4+\cfrac{1}{12+\cdots}}$

61. $8+\cfrac{1}{1+\cfrac{1}{16+\cdots}}$

62. $\frac{\sqrt{29}-5}{2}$. **64.** $\frac{\sqrt{13}-3}{2}$. **66.** $\frac{\sqrt{5}-1}{2}$. **68.** $\frac{\sqrt{15}-3}{2}$.

63. $\sqrt{5}-2$. **65.** $\sqrt{3}$. **67.** $\frac{3-\sqrt{5}}{2}$.

69. $\frac{\text{meter}}{\text{yard}}=\frac{1}{1};\ \frac{11}{10};\ \frac{12}{11};\ \frac{35}{32};\ \frac{292}{267};\ \frac{5875}{5372};\ \frac{6167}{5639}.$

70. $\frac{\text{fathom}}{\text{meter}}=\frac{1}{1};\ \frac{2}{1};\ \frac{9}{5};\ \frac{11}{6};\ \frac{64}{35};\ \frac{267}{146}.$

71. $\frac{\text{20 dollars}}{\text{20 francs}}=\frac{5}{1};\ \frac{26}{5};\ \frac{57}{11};\ \frac{482}{93};\ \frac{539}{104};\ \frac{1021}{197};\ \frac{6665}{1286}.$

72. $\frac{\text{eq. diameter}}{\text{pol. diameter}}=\frac{1}{1};\ \frac{299}{298};\ \frac{1795}{1789};\ \frac{3889}{3876};\ \frac{13462}{13417}.$

73. $\frac{\text{tropical year}}{\text{civil year}}=\frac{1}{1};\ \frac{1507}{1506};\ \frac{1508}{1507};\ \frac{3015}{3013};\ \frac{4523}{4520};$
$\frac{7538}{7533};\ \frac{12061}{12053}.$

Ex. 60. Page 208.

1. $1-6x+24x^2-96x^3$, etc. **2.** $\frac{1}{3}+\frac{1}{9}x+\frac{1}{27}x^2+\cdots$

3. $\frac{1}{3}+\frac{4}{9}a+\frac{16}{27}a^2+\frac{64}{81}a^3+\frac{256}{243}a^4+\cdots$

4. $\frac{1}{2} - \frac{5}{4}x + \frac{15}{8}x^2 - \frac{45}{16}x^3 + \cdots\cdots$

5. $1 + 2x + 3x^2 + 4x^3 + \cdots\cdots$

6. $1 - 2x + 3x^2 - 5x^3 + 8x^4 - \cdots\cdots$

7. $1 + 3x + 7x^2 + 17x^3 + 41x^4 + \cdots\cdots$

8. $1 + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \frac{1}{16}x^6 - \cdots\cdots$

9. $\frac{a}{b} + \frac{ac}{b^2}x + \frac{ac^2}{b^3}x^2 + \frac{ac^3}{b^4}x^3 + \cdots\cdots$

10. $a + \frac{b^2}{2a} - \frac{b^4}{8a^3} + \frac{b^6}{16a^5} - \cdots\cdots$

11. $\frac{1}{2x} + \frac{3}{4} + \frac{9x}{8} + \frac{27x^2}{16} + \frac{81x^3}{32} + \cdots\cdots$

12. $1 + \frac{3}{2}x + \frac{3}{8}x^2 - \frac{1}{16}x^3 + \frac{3}{128}x^4 - \cdots\cdots$

13. $\frac{x}{m} + \frac{x^2}{m^2} - \frac{x^4}{m^4} - \frac{x^5}{m^5} + \cdots\cdots$

14. $1 - \frac{3}{x} + \frac{5}{x^2} - \frac{7}{x^3} + \cdots\cdots$

15. $\frac{1}{3(x+2)} + \frac{5}{3(x-1)}.$

16. $\frac{2}{3+4x} + \frac{5}{6+7x}.$

17. $\frac{1}{2(x+1)} - \frac{1}{x+2} + \frac{1}{2(x+3)}.$

18. $\frac{9}{8x-7} + \frac{6}{5x-4} + \frac{3}{2x-1}.$

19. $\frac{Pa-Q}{(a-b)(1+ax)} - \frac{Pb-Q}{(a-b)(1+bx)}.$

20. $\frac{\frac{bc-ad}{cf-de}}{c-dx} + \frac{\frac{af-be}{cf-de}}{e-fx}.$

21. $\frac{1}{x-3}+\frac{3}{x+2}$.

22. $\frac{3}{2(x-1)}-\frac{7}{x-2}+\frac{13}{2(x-3)}$.

23. $\frac{1}{x^2}+\frac{1}{x}+\frac{1}{(x-1)^2}-\frac{1}{x-1}$.

24. $\frac{1}{3(x+1)}+\frac{5x-4}{3(x^2-x+1)}$.

25. $-\frac{2}{x^2}+\frac{3}{x}-\frac{3x-2}{(x^2+1)^2}-\frac{3x-2}{x^2+1}$.

26. $\frac{1}{ax}-\frac{1}{a(x-a)}+\frac{2}{(x-a)^2}$.

Ex. 61. Page 210.

1. 24, 24, 24.....

2. 0.

3. 4 ; 256.

4. 169 ; n^2.

5. $n(n+1)$; 156.

6. $\frac{1}{465}$.

7. 385 ; $\frac{n}{3}+\frac{n^2}{2}+\frac{n}{6}$.

8. 3150.

9. 364 ; $\frac{n}{6}(n+1)(n+2)$.

10. $\frac{n^5}{5}+\frac{n^4}{2}+\frac{n^3}{3}-\frac{n}{30}$.

11. $\log 512 = 2.709269$.

12. 3.6342411.

13. 10.414439.

14. $\frac{1}{x(x+2)}$; $\frac{3}{4}$.

15. $\frac{1}{\frac{x}{2}(x+1)}$; 2.

16. $\frac{11}{18}$.

17. $\frac{1}{(4x-3)(4x+1)}$; $\frac{1}{4}$.

18. $\frac{3}{5}$.

19. $\frac{n}{12+12n}$.

20. $\frac{1+4x+x^2}{(1-x)^4}$.

21. $\frac{13}{240}$.

22. 4760.

23. $\frac{1}{18}$.

24. $\frac{n}{3}(4n^2-1)$.

25. $\dfrac{1+2x-x^2}{1-2x+x^2-2x^3}$.

26. $\dfrac{1+2x+3x^2}{(1-x)^3}$; 6146.

Ex. 62. Page 212.

1. $x^3-6x^2+11x-6=0$.

2. $x^3-3x+2=0$.

3. $x^3-19x+30=0$.

4. $x^3+8x^2-5x-84=0$.

5. $x^3-7x^2-5x+35=0$.

6. $x^3-2x^2+3x-6=0$.

7. $x^3-8x-3=0$.

8. $6x^4-35x^3+62x^2-35x+6=0$.

9. $x^4+x^3-26x^2-44x+48=0$.

10. $2x^4-6x^3+3x^2-1=0$.

11. $\pm 3, \pm 2$.

12. $\pm 5, \pm 2\sqrt{-1}$.

13. $\pm\sqrt{1.5}, \pm\sqrt{-1.4}$.

14. 0, 1, 8.

15. 9, 5, 3, -1.

16. 0, 1, $\frac{1}{2}(1\pm\sqrt{22})$.

17. $\pm\frac{3}{2}, \pm\frac{1}{2}\sqrt{-19}$.

18. $\sqrt[3]{3}, \sqrt[3]{-\frac{9}{4}}$.

19. 5, $-4\frac{2}{7}$.

20. $\frac{1}{4}, -\frac{9}{8}$.

21. $\frac{1}{8}, -42\frac{7}{8}$.

22. 0, 2, -3.

23. 256, $\frac{625}{16}$.

24. $\sqrt[n]{\frac{9}{10}}, \sqrt[n]{-\frac{3}{2}}$.

25. $\frac{1}{2}(1\pm\sqrt{-2\pm\sqrt{3}})$.

26. 4, 1, $\dfrac{-5\pm\sqrt{-11}}{2}$.

27. $\pm 3\sqrt{\pm 1}, \pm 2\sqrt{\pm 1}$

28. 9, $\sqrt[3]{16}$.

29. 1, 4, 2, 3.

30. 5, 3, $4\pm\sqrt{10}$.

31. 4, $\sqrt[3]{\frac{1}{4}}$.

32. 2, -4, $-1\pm\sqrt{\frac{57}{2}}$.

33. 0, 3, $\frac{5}{7}, -2\frac{2}{7}$.

34. 4, $\sqrt[3]{25}$.

35. $7,\ -\frac{77}{25}$.

36. $4,\ 1,\ \frac{-3\pm\sqrt{-7}}{2}$.

37. $\pm\sqrt{c(2a-c)}$.

38. $\frac{4}{3},\ -\frac{1}{3},\ \frac{9\pm\sqrt{481}}{50}$.

39. $2,\ -5,\ \frac{1}{2}(3\pm\sqrt{-7})$.

40. $9,\ 4$.

41. $4,\ -2\pm\sqrt{-22}$.

42. $2,\ 3,\ 4$.

43. $3,\ \frac{-3\pm\sqrt{-7}}{2}$.

44. $-1,\ 2,\ 7$.

45. $3,\ -\frac{3}{2}\pm\frac{\sqrt{45}}{2}$.

46. $2,\ 5,\ 11$.

47. $-3,\ \frac{3\pm\sqrt{-7}}{2}$.

48. $1,\ 2,\ 3,\ 4$.

49. $5,\ \frac{-5\pm\sqrt{-83}}{2}$.

50. $1,\ -2,\ -6,\ 7$.

51. $1,\ -\frac{1}{2}(1\pm\sqrt{-3})$.

52. $1,\ 1\frac{2}{3},\ 2$.

53. $\frac{1}{5},\ -\frac{3}{10},\ -\frac{3}{2}$.

54. $-\frac{4}{3},\ \frac{2}{3}\pm\sqrt{15}$.

55. $\frac{1}{4},\ \frac{1}{3},\ \frac{1}{2}$.

56. 0.32748.

57. 3.135.

58. -0.6378.

59. 7.6172.

60. 1.42535.

61. $y^3-7y=90$.

62. $y^2-100=0$.

63. $y^3+6y-88=0$.

64. $y^3+6y-2=0$.

65. $y^3-\frac{31}{3}y-\frac{340}{27}=0$.

66. $y^4-\frac{17}{2}y^2+\frac{225}{16}=0$.

67. $4,\ 1\pm\sqrt{-1}$.

68. $1.1541,$
$-0.57708\pm1.9997\sqrt{-1}$.

69. $-4,\ 5\pm\sqrt{-3}$.

70. $-2,\ -5,\ -5$.

71. $10,\ \frac{11}{2}\pm\frac{5}{2}\sqrt{-3}$.

72. $3,\ \frac{1}{3},\ \frac{1}{3}(\pm\sqrt{55}-8)$.

73. $\frac{1}{2}(1\pm\sqrt{-3}),\ 1,\ 1.$
$\frac{1}{2}(3\pm\sqrt{5})$.

74. $1,\ 1.56155,\ -2.56155$.

75. $-1.18479,\ -2.22668,$
-3.41147.

76. 11.00675.

77. $-\frac{1}{2},\ -\frac{2}{5},\ -\frac{3}{7},\ -\frac{5}{12},\ -\frac{23}{55},$
$-\frac{120}{287},\ -\frac{143}{342}$.

78. $4,\ -2,\ -2$.

Ex. 63. Page 215.

1. 5.
2. 1.19897.
3. 0.4342945.
4. 3.86633.
5. 2.718281.
6. 5.
7. -0.991318 .....
8. $2\frac{1}{4}$.
9. -8.
10. 0.35425.
11. 7.

12. $\dfrac{m \log b - n \log a}{m \log a - n \log b}$

13. -1.553174.

14. $x = 3$; $y = 5$.

15. $$x = \frac{\log b \log q - \log d \log p}{\log b \log c - \log a \log d};$$
$$y = \frac{\log c \log p - \log a \log q}{\log b \log c - \log a \log d}.$$

16. $x = 3$; $y = 4$.

17. $\dfrac{\log(b \pm \sqrt{b^2 - 4}) - \log 2}{\log a}$.

18. 3.597.

19. 3.2164.

20. 1.38736.

21. $x = 3$; $y = 5$.

22. $x = 7$; $y = 121$.

23. $x = 2$; $y = 3$; $z = 2$.

24. 7 or 4.

26. 7 or -12.

PART SECOND.

ANSWERS TO EXAMINATION MANUAL.

1.

1. 94.
2. $10a-b+3c$.
3. $1-x^3$.
4. $xy(a^2x-b^2y)$.
5. $xy(x^2-y^2)$.
6. $\frac{1}{6}$.
7. 9.
8. 2 inches; 10 inches.

2.

1. -4.
2. $a+2b-c$.
3. $a^2(x-y)(x+y)$.
4. $3x+5$.
5. $\frac{2(a-c)}{ac}$.
6. 1.
7. A \$2400; B \$2500.
8. $x=5,\ y=4$.

3.

1. 2.
2. $x^4-\frac{a^4}{16}$.
3. $2a-\frac{1}{2}x$.
4. $2a^3(a-x)$.
5. $\frac{a+x}{ax}$.
6. 7.
7. A 36 years;
   B 18 years.
8. $x=2,\ y=1$.

4.

1. $a^2-b^2+2bc-c^2$; 80.
2. $a+9c+b-14d-6e$.
3. $x^5+x^4y+x^3y^2+x^2y^3$
   $+xy^4+y^5$.
4. 72.
5. $x=13,\ y=7$.
6. $(y+30)(y-5)$.
7. $\frac{(a-2)(a-5)}{a^2}$.
8. 420.

5.

1. $3xy - 3bx + 3ay - 2ab$
$- ac + ad.$
2. $4x^2(x-3).$
3. $12a^2b^2(a^2-b^2)^3.$
4. $\frac{x^2-4}{x^2-9}.$
5. $\frac{1}{2}.$
6. $\$26\frac{2}{3}.$
7. 2h. 8m. $34\frac{2}{7}$s.

6.

1. $a+b+c.$
2. $2a^3 - 11a^2b - 2ab^2$
$+ 2b^3 + 8.$
3. $x^5 - x^3y^2 + x^2y^3 - y^5.$
4. $a^2 + ab + b^2 + ac - bc + c^2.$
5. 4.
6. $\frac{2x^2+1}{x^2-1}.$
7. $x=3,\ y=2,\ z=1.$
8. $900.

7.

1. $40\frac{3}{4}.$
2. $x.$
3. $1 + 6x + 16x^2 + 28x^3$
$+ 27x^4 + 18x^5.$
4. $x^4 + 2x^3y - 2xy^3 - y^4.$
5. $x^2 + (b+c)x + bc.$
6. $\frac{a+b}{a-b}.$
7. 8.
8. 20 years.

8.

1. $1\frac{2}{9}.$
2. 60.
3. $xy^2 - x - x^2 + y^2.$
4. $mpx^3 + (mq - np)x^2$
$- (mr + nq)x + nr.$
5. $1 + x^2 + 4y^2 - x + 2y + 2xy.$
6. 1.
7. 320 yards by 180 yards.

9.

1. 27.
2. $x^{2a} - 2x^ay^b + 2y^{2b}.$
3. $a(x-1)(x+1);$
$(x^2+9a^2)(x+3a)(x-3a);$
$(2a+3)(4a^2-6a+9);$
$(x+5)(x+2).$
4. $(a^2-b^2)(b^2+c^2)(c^2-a^2)$
$(b-c).$
5. $x - y + z.$
6. $-x - 2y + 4.$
7. $\frac{a(6a-b)}{6b-a}.$
8. $46\frac{1}{2}$ and $23\frac{1}{2}.$

10.

1. 25.
2. $3x^3-3x^2+3x-3$.
3. $2a^{3n}-2x^{3n}$.
4. x^2-3x-y.
5. $(x^8+y^8)(x^4+y^4)(x^2+y^2)(x+y)(x-y)$;
   $\left(2x^2+\frac{1}{x}\right)\left(4x^4-2x+\frac{1}{x^2}\right)$;
   $(4x^6+y^2)(2x^3+y)(2x^3-y)$;
   $(x+7)(x-1)$.
6. $2a+3x$.
7. $\dfrac{2ab}{a+b}$.
8. A 40 years; B 20 years.

11.

1. 10.
2. $2x^4+\frac{1}{2}x^2+\frac{9}{2}x+\frac{1}{2}$.
3. $1+2x+3x^2+4x^3+5x^4$.
4. $-6x^2$.
5. $(2x-5)(x+1)$;
   $(a+b-c)(a-b-c)$.
6. y^2+my+m^2.
7. 7.
8. \$3.75.

12.

1. $3a+b$.
2. $-x-2y-4$.
3. $3a^2-2ab+5b^2$.
4. 66 years.
5. $x^4-\dfrac{9x^2y^2}{16}+\dfrac{3xy^3}{2}-y^4$.
6. 5.
7. $\frac{3}{15}$.
8. $x=7,\ y=8,\ z=9$.

13.

1. $11x^3-5x+4y$.
2. $2x+6a+13b-7c-13$.
3. $27x^3-36x^2+6xy+24x$
   $-y^3-4y^2-8y-8$.
4. $x-2$.
5. 13.
6. $(z+x-y)(z-x+y)$.
7. $\dfrac{x+y}{y}$.
8. $x=1,\ y=-1,\ z=2$.

14.

1. $x^2 + x - \frac{1}{2}$.
2. $x^2 - 2$.
3. $(x + 7)(x + 13)$.
4. 52, 34.
5. $\frac{b}{a - b}$.
6. $\frac{a^7(x^2 - a^2)}{x^2(x^2 + a^2)}$.
7. $x = 35,\ y = 30,\ z = 25$.

15.

1. $a^5 - 4a^4b + 4a^3b^2 + 4a^2b^3 - 17ab^4 - 12b^5$.
2. $x^2 - ax + b^2$.
3. $\frac{2a}{b} + \frac{2b}{a}$.
4. $2\frac{4}{9}$.
5. $x = 4,\ y = 0,\ z = 5$.
6. $(x - z - a + y)(x - z + a - y)$.
7. 0.

16.

1. $a - b$ feet; north; south.
2. $8x - 8$.
3. $a + 2b - c$.
4. $a^4(a - 3b^3)(a + 3b^3)$.
5. $2(x + a)$.
6. $\frac{y}{x + y}$.
7. 3.
8. \$3, \$5, and \$7.

17.

1. $(a + b - c)(a - b + c)(a + b + c)(a - b - c)$.
2. $x(2x - y)$.
3. $5b - 5c$.
4. $x^4 - x^2y^2 + y^4$.
5. $x = y = 5$.
6. 49, 42.
7. $4x^2 - 4x + 1$.

18.

1. $(a - b + c)x^2 + (a + b - c)xy - (a - b - c)y^2$.
2. $x^4 - (a^2 + b^2)x^2 + a^2b^2$.
3. $(2x + y)(16x^4 - 8x^3y + 4x^2y^2 - 2xy^3 + y^4)$;
   $(5x - 7y)(x + 3y)$; $(3ax + 1)(2ax - 3)$.
4. a.
5. $30x(1 - x)^2(1 + x)$.
6. $x - 1$.
7. $2\frac{2}{3}$.
8. 24 days.

19.

1. $ax^{m+n}+ax^{m}-(a-b)x^{m-n}+ax^{m-2n}$.
2. $-(3ab+b^2)$.
3. $\dfrac{a+b+c}{a-b+c}$.
4. 0.
5. $a-3b$.
6. $\dfrac{4a^2-3ab-b^2+4a}{4a+b}$.
7. $\dfrac{(a-2b)(2a-b)}{a}$.
8. 64 square yards.

20.

1. $2a^{2n}-4a^nb^n+2b^{2n}$.
2. $x(5x^2-1)$.
3. -1.
4. $(2x-1)(2x+1)(4x^2+1)$ $(16x^4+1)$ and $(x-9)(x+2)$.
5. 1. 6. $\dfrac{ac}{b}$. 7. 28.

21.

1. $(a-x)^2(a+x)$; $(x-2y)(x+2y)(x+3y)$.
2. $\dfrac{1-x}{x(2x-1)(2x+1)}$.
3. $\dfrac{a+b}{ab(a-b)}$.
4. 1.
5. $8x^3-14x+6$.
6. $7+x$.
7. $240.

22.

1. $(a^2x^2+\frac{9}{4}y^2)(ax+\frac{3}{2}y)(ax-\frac{3}{2}y)$;
   $bx(3x-by)(9x^2+3bxy+b^2y^2)$;
   $2a(5x-3a)(2x+3a)$.
2. $\dfrac{x^2+1}{x^2-1}$.
3. $(x+2y)(x-2y)(x+3y)$.
4. 7.
5. 10. 6. 24 days.

23.

1. -1.
2. $\frac{7}{6}a-\frac{1}{6}b+\frac{1}{4}c-\frac{1}{12}d$.
3. $x^4+2ax^3+3a^2x^2+2a^3x+a^4$.
4. $\dfrac{x^2-ax+a^2}{x+a}$.
5. $x=\frac{3}{2}$, $y=\frac{2}{3}$.
6. $x^2-x+\frac{1}{4}$.
7. After 8 hours; 38 miles from the starting-place.

24.

1. 42.
2. x^2-6x+9.
3. $2ab$.
4. $4x(x-y)$.
5. $x=7,\ y=5$.
6. 104.

25.

1. $(x^4-x^2y^2+y^4)(x^2+xy+y^2)(x^2-xy+y^2)(x^2+y^2)(x+y)(x-y)$.
2. $a^2-3b^2+3c^2$.
3. $2x^2(2x+3a)$.
4. $\dfrac{a+c}{a-c}$.
5. $\dfrac{4a-b}{2a-3c}$.
6. $x=1,\ y=\frac{1}{3},\ z=\frac{1}{5}$.
7. 75.

26.

1. $x=4,\ y=6,\ z=8$.
2. $2\frac{1}{2}$ miles an hour.
3. x^2-2x+1. 4. 21.
5. $a^2-2ax+bx-x^2$.
6. $\dfrac{2x}{x+y}$.

27.

1. $2xy^3-(3x^2-a)y^2$
   $-(x^3-a^3)y+2a^2x^2$.
2. $\dfrac{a^2-4b^2}{a}$.
3. $a+b$.
4. $\dfrac{x}{x^2+y^2}$.
5. 25 and 26.
6. $134\frac{1}{6}$ miles.
7. $x=24,\ y=12$.

28.

1. $x^3+x^5+x^7$; $\dfrac{x^9}{1-x^2}$.
2. $6x^4-(3a^2+12)x^3$
   $-4ax+3a^3$.
3. $a-y$.
5. $\dfrac{x+a}{2a}$.
6. A 62 h.; B 93 h.; C 155 h.
7. $x=5,\ y=2$.

29.

1. $\frac{a^2+x^2}{2ax}$.
2. $\frac{1}{1-x+x^2}$.
3. $x=3,\ y=4$.
4. $x(x^2-2x+1)$.
5. $x,\ x-2y,\ x+2y,\ x(x-2y)$ $x(x+2y),\ (x-2y)(x+2y)$ $x(x-2y)(x+2y)$.
6. 52.
7. 8.

30.

2. a^2+b^2.
3. $\frac{a^2}{x}-\frac{x^3}{a^2}$.
4. $\frac{a}{b}$.
5. $3a-b$.
6. $x=1,\ y=3,\ z=5$.
7. 35; 14.

31.

2. $x=\frac{m^2-n^2}{ma-nb},\ y=\frac{n^2-m^2}{na-mb}$.
3. $x=2,\ y=3,\ z=4$.
4. $x+3$.
5. $\frac{(x+y)(x^2+xy+y^2)}{y}$.
6. $\frac{2}{7}$.
7. $(1+b+a^2)x^3-(c^2+c+1)x^2$ $-(a+b+c)x$.

32.

2. $2(2x^2+7x+3)$.
3. 1.
4. $\frac{ac}{b}$.
5. $x=10,\ y=6$.
6. $x=7,\ y=9,\ z=11$.
7. 588 and 708.

33.

1. $-\frac{2a^2}{b^2+2b}$.
2. $x=\frac{ac+d}{b+c},\quad y=\frac{ab-d}{b+c}$.
3. $x=5,\ y=4,\ z=3$.
4. 0.
5. $\frac{7x}{6a}$.
6. $x^4+2ax^3+3a^2x^2+2a^3x+a^4$.
7. $-2b(3a+b)(a^2-2b^2)$.
8. 25; 15.

34.

2. $4b^2(a+x)^2$;
$216a^2b^6x^2(a^2-x^2)^3$.
3. $a^{10}-1$.
4. $\dfrac{2+2x+x^2}{3+5x+3x^2+x^3}$.
5. $\dfrac{a^3+b^3}{a^4b^2+a^2b^4}$.
6. $x=6,\ y=4$.
7. \$140, \$60, \$70, \$20.
8. $32x^{10}y^5z^{15}$, $-343a^9b^{12}c^{15}$,
$81a^4b^{16}c^8$.

35.

1. 0.
2. $\dfrac{a+b}{a-b}$.
3. $x=y=z=12$.
4. $144a(a^2-9)$.
5. $x^2-4x-21$.
6. 10 P.M.; half way.

36.

1. 0.
2. $ax^2-bxy+cy^2$.
3. $a-b$.
4. $(a^2+x^2)(a^3+x^3)(a^3-x^3)$.
5. $\dfrac{a+2}{2}$.
6. A's age 45; B's age 20.
8. $\dfrac{a^3}{b^3}+3\dfrac{a}{b}+3\dfrac{b}{a}+\dfrac{b^3}{a^3}$.

37.

1. 1.
2. $\dfrac{ab(a+b)^2}{a^2+ab+b^2}$.
3. $x=4,\ y=5,\ z=6$.
4. $24\frac{1}{2}$ by $12\frac{1}{2}$.
5. $x^2-\frac{1}{2}x+\frac{1}{2}$.
6. $\dfrac{x^6}{y^6}-3\dfrac{x^3}{y^3}+3-\dfrac{y^3}{x^3}$.

38.

1. $\dfrac{8x^3}{a^3}-\dfrac{27y^3}{b^3}$.
2. $x+3$.
3. $\dfrac{14x+4}{x^2-1}$.
4. $x^2+\frac{1}{3}x+\frac{1}{2}$.
5. $\dfrac{a}{a-b}$.
6. A \$3700; B \$3300.
7. $8a^3-36\dfrac{a^2}{x}+54\dfrac{a}{x^2}-\dfrac{27}{x^3}$.

39.

1. $\frac{1}{abc}$.
2. 24.
3. $x = 8,\ y = 12$.
4. $\frac{a}{3b} - \frac{3b}{a}$.
5. $8x - 8$.
6. $bc(a^2 - 1)$.
7. A 116; B 109; C 114.

40.

1. $3b(2a^3 - a^2b - 2ab^2 + b^3)$.
2. $3a + b$.
3. $8(1-x)^2(1+x)(1+x^2)$.
4. $1 - 2x + 3x^2$.
5. $\frac{1}{4}$.
6. $x = 6,\ y = 3,\ z = 8$.
7. $43\frac{7}{11}$ min. past 2 o'clock.

41.

1. $1 - \frac{1}{2}xy - 2x^2y^2$.
2. $\frac{1 + 5a}{3(1 + 2a)}$.
3. $x = 3,\ y = 2,\ z = 1$.
4. $\frac{x^3}{3} - \frac{17x^2}{36} + \frac{x}{3} - \frac{1}{8}$.
5. $2\frac{2}{45}$ miles from one end; $1\frac{43}{45}$ miles from the other.
6. $\frac{a^3}{8} + \frac{a^2b}{2} + \frac{2ab^2}{3} + \frac{8b^3}{27}$.
7. $3a$.

42.

1. $\frac{18}{x^2 - 9}$.
2. $x - y$.
3. $a - \frac{b}{3}$.
4. $\frac{3}{4}$.
5. $x = 1\frac{5}{9},\ y = \frac{1}{2},\ z = 1\frac{7}{18}$.
6. $2 + 4\sqrt{a}$.
7. $13\frac{1}{2}$ inches.

43.

1. $(9x^2 - x - 3)(x + 6)(x + 5)$.
2. $x^2 - 2x - \frac{3}{2}$.
3. $x = 10,\ y = 8,\ z = 6$.
4. $\frac{5}{2}x^2 + 3ax - \frac{7}{3}a^2$.
5. $\frac{4}{3(x + 1)}$.
6. $\frac{yz + xz + xy}{yz + xz - xy}$.
7. $\frac{x^2 - x}{2x - 1}$.

44.

1. $\frac{2}{(2x+1)(2x-1)^2}$. 2. $\frac{16ax}{(a+x)^2}$.
3. $\frac{5a^2}{12}+\frac{5b^2}{12}+\frac{5c^2}{12}$.
4. $(x^2+y^2)(x+y)(x-y)$.
5. $a+2b+c$.
6. $a+b$.
7. 18, 22, 10, 40.

45.

1. $\frac{2}{(1+a)(1-4a^2)}$.
2. $-1+3x^2-x^4$.
3. $x+2$.
4. $(1+x)(1-x)^2(1+x^2)$.
5. -1.
6. $\frac{a^3b^3}{64}-\frac{3ab}{16}+\frac{3}{4ab}-\frac{1}{a^3b^3}$.
7. $x=\frac{1}{3}$, $y=\frac{1}{4}$, $z=\frac{1}{5}$.

46.

1. $\frac{ax^3-ax}{a^2x-a^2-x^2}$.
2. $\frac{x-3}{2x^2+5}$.
3. $\frac{2x^2-3xy-2y^2}{4xy}$.
4. $\frac{4a^2-3b}{8a+3ac-3}$.
5. Father's age, 50; son's age, 25.
6. $x=17$, $y=19$.
7. $\frac{2x(1+ax^2)}{2+ax-2x^2}$.

47.

1. $a^{10}-1$.
2. $2(2x^2+7x+3)$.
3. $(x^3-1)(x^3+1)(x+1)(x-1)$.
4. $x=2$, $y=1$, $z=3$.
5. \$$3\frac{1}{8}$.
6. 42.

48.

1. $\frac{(x+1)^2}{2}$.
2. $x-3$.
3. $\frac{5x}{(x+1)(2x-1)(3x+1)}$.
4. $\frac{2x+1}{5x+3}$. 5. $\frac{x+2}{x+3}$.
6. $\frac{a+m^2}{b+n^2}$; 0.

49.

1. c.
2. $x=\dfrac{cb'-bc'}{ab'-ba'}$, $y=\dfrac{ca'-ac'}{a'b-ab'}$.
3. $x=5$, $y=-5$, $z=5$.
4. $x=\dfrac{2}{m+p-n}$, $y=\dfrac{2}{m+n-p}$, $z=\dfrac{2}{n+p-m}$.
5. $x=12$, $y=30$, $z=168$.

50.

1. After 8 hours; 38 miles from the starting-place.
2. 860.
3. A \$8; B \$4.
4. 2 hours 24 minutes; $\dfrac{mnp}{mn+mp+np}$ hours.

51.

1. $(a-b+c)x^3-(b-c+d)x^2-(c+d+e)x$.
2. $a^2+b^2+c^2+ab-ac+bc$.
3. $\dfrac{a+bx}{b+ax}$.
4. 69; 81.
5. $\pm\frac{2}{3}$.
6. 3, or -1.

52.

1. $2b(x+y)$.
2. 22.
3. $3(ax+2)$.
4. $\pm\dfrac{a}{8}$.
5. 3, or $-\frac{4}{5}$.
6. 40 yds.; 24 yds.

53.

1. $24a^2b^2(a^2-b^2)$.
2. $\dfrac{2a^3}{a^4-x^4}$.
3. $\dfrac{1}{32c^{\frac{12}{5}}}$.
4. $8x^{\frac{3}{4}}+4x^{\frac{1}{2}}y^{\frac{1}{2}}+2x^{\frac{1}{4}}y+y^{\frac{3}{2}}$.
5. 7, or -2.
6. 25; 20.

54.

1. $x = -5,\ y = 6,\ z = -2.$
2. \$180; \$150.
3. $\frac{a}{b^{\frac{2}{3}}}.$
4. $x^{-\frac{2}{3}} + x^{-\frac{1}{3}}y^{-\frac{1}{3}} + y^{-\frac{2}{3}}$
5. $\sqrt[5]{\frac{1}{18}}.$
6. 5, or $-4\frac{1}{3}.$

55.

1. $36xy^2(x^2 - y^2).$
2. $\frac{x - 3x^2 + 3x^3}{(1-x)^3}.$
3. $3\sqrt[3]{7};\ 4\sqrt{2};\ 2\sqrt[3]{22}.$
4. $x^2 - ax - a^2.$
5. $18\sqrt[3]{2}.$
6. 1, or $10\frac{2}{3}.$

56.

2. $2ax\sqrt[6]{72ab^5}.$
3. $13\sqrt[3]{2}.$
4. 1.
5. $2\frac{1}{2}$, or $-4\frac{7}{10}.$
6. 358 yards.

57.

1. $x^{-4}y^4.$
2. $\sqrt{2}.$
3. \$60, or \$40.
4. -4, or $-21.$
5. $x = 4,\ y = -5,\ z = 6.$
6. 1540; 880; 616.

58.

1. 13.
2. $a(a^2 - 1)(a + 3)(a - 6).$
3. 147 miles.
4. $\frac{1}{3}\sqrt[4]{27}.$
5. 6, or 3.
6. A \$1800; B \$1600.

59.

1. $x = \frac{ac + b^2}{a^2 + b},\ y = \frac{ab - c}{a^2 + b}.$
2. $-\frac{2}{7}.$
3. $x^2 - 2.$
4. $\frac{2y^3}{x(x^2 - y^2)}.$
5. $a - 2x + 3.$
6. $54\frac{6}{11}$ minutes past 10.
7. 4, or $-1.$

60.

1. 8.
2. $\frac{(x+y)(x^2+xy+y^2)}{y}$.
3. x^2+7.
4. $x+\frac{x}{y}+1$.
5. 12.
6. $\frac{3}{7}$.
7. 2 miles an hour.

61.

1. 2, or $-\frac{8}{7}$.
2. $\frac{2}{a}$.
3. $x=a$, $y=b$.
4. $12x-5$.
5. $a^2+b^2+c^2-ab-ac-bc$.
6. $x^2-x+\frac{1}{4}$.
7. The first pipe 6 minutes, and the second pipe 4 minutes.

62.

1. $6a^2+25ab-13ac-b^2+c^2$; 100.
2. $x^4+x^2+1+\frac{1}{x^2}+\frac{1}{x^4}$.
3. $\frac{(m+n)^2}{2n}$.
4. $4x-1$.
5. $a-2b^2-3c^3$.
6. $x=5$, $y=6$.
7. 1, or $\frac{85}{81}$.
8. $32\frac{8}{11}$ minutes past 3.

63.

1. $3\frac{1}{2}$.
2. $x=3$, $y=2$, $z=1$.
3. 1, or $\frac{28}{27}$.
4. $x^4-7x^2+26x-40$.
5. b.
6. $\$125$.

64.

1. $\frac{18x^2+26x-14}{2(x+2)(2x+1)}$.
2. $\frac{3}{4}x^3-\frac{1}{2}x^2+1$.
3. $(a^2-b)(a^2+b)(a-b)$.
4. $a+\frac{b}{2a}-\frac{b^2}{8a^3}$, etc.
5. $\frac{a+b-2c}{2(a+b)}$.
6. $3\frac{3}{4}$, or -4.
7. 41; 14.

65.

1. $\frac{4}{3x}$.
2. $x^2(x^2-1)(x^2-4)$.
3. 3.
4. $x=5,\ y=4,\ z=3$.
5. $2x$.
6. b, or $-\frac{ab}{a+b}$.

66.

1. 520; 480.
2. $\frac{2x^2+1}{3y^3+y}$.
3. $5x^2+1$.
4. $m+1-\frac{1}{m}$.
5. $x=\frac{3a}{2},\ y=\frac{3b}{2}$.
6. $\frac{1}{2}$, or -1.

67.

1. $y=\frac{2x}{1+x^2}$.
2. 1.
3. a^{pq+qr}.
4. $\frac{c^2-ab}{a+b}$.
5. $x=11,\ y=1$.
6. 4 miles an hour.

68.

1. $a-b;\ -\frac{a^2+ab+b^2}{a}$.
2. $\frac{1}{3}x^3+\frac{11}{6}x^2-\frac{7}{9}x+\frac{4}{3}$.
3. x^2+5x+6.
4. $\frac{x}{x^2+y^2}$.
5. $x=6,\ y=15$.
6. $\frac{a^3}{27}-\frac{ab}{c}+\frac{9b^2}{ac^2}-\frac{27b^3}{a^3c^3}$.

69.

1. $x=a$, or $x=b$, or $x=c$, or $x=d$.
2. $(m+n+p+q)(m-n-p+q)$. **3.** 12.
4. A \$5000; B \$5000; C \$10,000. **5.** $a-y$. **6.** $a+b$, or 0.

70.

1. $\frac{13}{14}$.
2. $(a^2-2bc)^2$.
3. $2x^2-3x+4$.
4. 1.
5. $\dfrac{2(x+2)}{(x^2-1)(x^2+x+1)}$.
6. $x=\frac{1}{2},\ y=\frac{1}{3},\ z=\frac{1}{4}$.
7. $3\frac{3}{4}$, or -4.

71.

1. $\frac{37}{8}x^2-\frac{31}{12}x$.
2. $\dfrac{1}{1-x^4}$.
3. $a+b+c$.
4. 750 English; 375 French.
5. $\dfrac{2a+b}{2a-b}$, or $\dfrac{2a-b}{2a+b}$.
6. 4 miles an hour.

72.

1. 1.
2. $x+\dfrac{y}{4x}-\dfrac{y^2}{32x^3}$, etc.
3. $\dfrac{a^2}{c\sqrt[12]{b}}$.
4. $x=\dfrac{b-c}{b+c-2a},\ y=\dfrac{c-a}{b+c-2a},\ z=\dfrac{a-b}{b+c-2a}$.
5. -4, or -21.
6. $x-2$.

73.

1. $\sqrt[6]{c}$.
2. $(x-4)(x-5)(x+11)$.
3. $\frac{7}{36}$.
4. 8, 6.
5. $x=\pm\sqrt{m^2+n^2}$.
6. $4x^2+6ax+a$.
7. $-8\frac{1}{3}$.

74.

1. $12\frac{4}{13}$ yards.
2. 26; 25.
3. 2.
4. $x+1$.
5. 1.
6. $\frac{1}{2}$, or $-\frac{1}{3}$.
7. $\left(\dfrac{b}{a}\right)^{\frac{3}{4}}$.

75.

1. $-\frac{1}{22}$.
2. $\left(\frac{1}{a^3}, a^{-3}, \frac{a^4}{a^7}\right)$, $(a^2, \sqrt{a^4}, a^{\frac{4}{2}}, a^3 \times a^{-1})$, $(a^{\frac{1}{2}}, \sqrt{a}, \sqrt[6]{a^3})$.
3. $x^2 + 1 + \frac{1}{x^2}$.
4. $x = 111$, $y = 77$.
5. $a^2 b^4$.
6. 15 miles; 2 miles an hour.
7. 4 miles an hour.

76.

1. $x^{\frac{5}{2}} - x^2 - 4x^{\frac{3}{2}} + 6x - 2x^{\frac{1}{2}}$.
2. $\frac{65}{a}$.
4. $\frac{x}{y}$.
5. $24\sqrt{3}$.
6. $a^4 b^{\frac{1}{2}}$.
7. $16\frac{3}{4}$, or $20\frac{3}{4}$.

77.

1. 4 hours.
2. $2a$, or $2b$.
3. $\frac{a}{2}(-1 \pm \sqrt{5})$.
4. 36 feet a second; 30 feet a second.
5. $\frac{2}{3}$, or $\frac{1}{3}$.

78.

1. $a^3 + a - a^{-1} - a^{-3}$.
3. 4.
4. $\frac{a^{\frac{2}{3}} + b^{\frac{11}{12}}}{a^{\frac{1}{3}} b^{\frac{2}{3}}}$.
5. A $160; B $400.
6. $x = \pm 5$, $y = \pm 3$.

79.

1. $\frac{3}{5}$.
2. $(a-1)(a+1)(a-2)(a+2)$.
3. $\frac{(a-b)^2}{2b}$.
4. 7; 8.
5. $134\frac{1}{6}$ miles.
6. $\frac{1}{2a}(-b \pm \sqrt{b^2 - 4ac})$.
   $a = c$.

80.

1. 100 lbs.
2. $-4480 a^3 b^{13}$.
3. $4m^2 + \frac{2}{3}m + 1$.
4. $x - a$.
5. $5xy^2z^3$.
6. $\frac{b-a}{2b-a}$, or $-\frac{b+a}{2b+a}$.
7. 5; 2.

81.

1. c.
2. $x^2+y^2+z^2$.
3. 20.
4. 4.
5. $x=6,\ y=5$.
6. 12; 13.
7. $(x+1)(x-2)(x+3)(x+4)$.

82.

1. 4.
2. 10.
3. a^6-b^6.
4. x.
5. $-\frac{b}{2a}\pm\frac{1}{2a}\sqrt{b^2-4ac}$. The roots are real, equal, or imaginary, according as $b^2>4ac$, $b^2=4ac$, or $b^2<4ac$.
6. $4x^2\sqrt[3]{xy^2}$.
7. $x=y=2$, or $-\frac{1}{2}$.

83.

1. $x-1$.
2. $\frac{63a}{250c}\sqrt[60]{a^4b^{45}}$.
3. $3\frac{1}{9}$ days.
4. $\frac{3}{a}(2\pm\sqrt{a^2+4})$.
5. 3.
6. $a^6-18a^5x+135a^4x^2-540a^3x^3+1215a^2x^4-1458ax^5+729x^6$.

84.

2. $\frac{9x-1}{1-9x^2}$.
3. 16 miles.
4. $(x^4+\frac{1}{16})(x^2+\frac{1}{4})(x+\frac{1}{2})(x-\frac{1}{2})$.
6. $x=\frac{1}{2},\ y=\frac{1}{3}$.
7. $2a-b$, or $a+2b$.

85.

2. $\frac{2+3x}{1+5x}$.
3. $\frac{40}{b^4}\sqrt[10]{\frac{a^7}{b^6}}$.
4. ± 3, or ± 1.
5. $5-2\sqrt{2}$.
6. $1-\frac{15y}{4}+\frac{45y^2}{8}-\frac{135y^3}{32}+\frac{405y^4}{256}-\frac{243y^5}{1024}$.

86.

1. $2a + 6b$, or $a - 8b$.
2. $\frac{1+x}{1+x^2}$.
3. 2.
4. $16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$.
5. 7 hours; 5 hours.

87.

1. 9.
2. 16.
3. 1.
4. $\frac{c(a-b)}{a}$.
5. $32\frac{8}{11}$ minutes past 6.
6. 3 inches.

88.

1. $\frac{12 \times 11 \times 10}{1 \times 2 \times 3}(2x)^9(-5y)^3 = -14080000x^9y^3$.
2. $\frac{a}{2x^2}$.
3. $x = \frac{c}{2a}(a+b)$, $y = \frac{c}{2a}(a-b)$.
4. The carriage, $300; the horse, $375; the harness, $45.
5. $\frac{a}{2} - 3b$, or $-(5a+b)$.

89.

1. ± 2, or ± 1.
2. 0, or a.
3. $-\frac{1}{8}a^{\frac{1}{3}}b^{-\frac{1}{3}}c^{\frac{2}{3}}$.
4. $x = a$, $y = 0$.
5. 300.

90.

1. $\pm 9a^{-\frac{2}{3}}b^1c^{\frac{1}{4}}$.
2. $\sqrt[3]{3}$.
3. $a^{\frac{1}{4}}$.
5. 6 rods.
6. $x = 3$, or $3\frac{7}{8}$; $y = 2$, or $1\frac{1}{8}$.

91.

1. $\frac{a^8b^{\frac{2}{3}}x^6y^{\frac{1}{2}}}{16}$.
2. Rate, 20 miles; distance, 140 miles.
4. $\frac{1}{a^2}$, or $\frac{1}{4a^2}$.
5. $\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} \times 3^5 \times 4^4 \times x^{\frac{5}{3}} \times y^2$.

92.

1. $\frac{7}{2}a$.
2. $\dfrac{-y^3}{2x(x-y)^2(x+y)}$.
3. $x^{-\frac{2}{3}}y^{\frac{1}{3}}$.
4. $\left(\dfrac{2ab}{b-2a}\right)^2$.
5. $x=2$ or $\frac{1}{3}$, $y=2$ or $\frac{2}{3}$.
6. 5 miles an hour.

93.

1. $(c-a)\sqrt{5m}$.
2. $2x(x^2+xy+y^2)$.
3. $\dfrac{2a-b}{ac}$, or $-\dfrac{3a+2b}{bc}$.
4. 72 feet.
5. The former.
6. $x=4$, or 2; $y=2$, or 4.

94.

1. $x=\pm 2$, or $\pm\sqrt{2}$; $y=\pm 4$, or $\pm 3\sqrt{2}$.
2. $-\dfrac{35x}{y}$; $\dfrac{35y}{x}$.
3. 5, or $5\frac{1}{2}$.
5. $x^2-13x+42=0$; 6, or 7.
6. 84; 48.

95.

1. $m=8$.
2. 5.
3. 25; 20.
4. 3, or $\frac{3}{2}(-1\pm\sqrt{-3})$, or $-\sqrt[3]{7}$.
5. $\dfrac{y^{\frac{1}{3}}}{(bx)^{\frac{1}{3}}}$.
6. $x-1$.

96.

1. $\dfrac{x^2+4y^2+z^2+2xy+xz-2yz}{x^2}$.
2. xy^2; $918a^3b^3x^5y^4$.
3. $x^2+\dfrac{1}{x}-\dfrac{1}{3}$.
4. $x=\pm\frac{3}{2}$ or $\pm\sqrt{\frac{1}{2}}$, $y=\pm\frac{1}{2}$ or $\mp\sqrt{2}$.
5. Each root $=-\dfrac{b}{a}$ if $b^2=ac$.
6. $1\frac{1}{3}$.

97.

1. $x^{\frac{5}{2}}y^{\frac{5}{2}}-\frac{5}{2}x^2y^{\frac{3}{2}}z+\frac{5}{2}x^{\frac{3}{2}}y^{\frac{1}{2}}z^2-\frac{5}{4}xy^{-\frac{1}{2}}z^3+\frac{5}{16}x^{\frac{1}{2}}y^{-\frac{3}{2}}z^4-\frac{1}{32}y^{-\frac{5}{2}}z^5$.
2. $\frac{89+28\sqrt{10}}{9}$; 19.726..... 3. 4, 1, $3\frac{2}{3}$, or $1\frac{1}{3}$.
4. Distance, 15 miles; and the rates of walking, 3, $2\frac{1}{2}$, and 4 miles.

98.

1. $\frac{1}{3}(\sqrt{2}+\sqrt{3}+\sqrt{5})$.
2. $\frac{1}{1-x^2}$.
3. 1, or $\frac{1}{21}$.
4. a^2-b^2.
5. $x=5$, or $\frac{3}{4}$; $y=3$, or $-1\frac{1}{4}$.
6. $16x^3\sqrt[5]{x}-16x^2y^2\sqrt[5]{x^2}+6xy^4\sqrt[5]{x^3}-y^6\sqrt[5]{x^4}+\frac{y^8}{16}$.

99

1. 0.
2. $1+2y^{-\frac{1}{3}}-3y^{-\frac{2}{3}}+4y^{-1}$.
3. $\frac{16x^4}{y^8}-\frac{32x^{\frac{7}{2}}}{3y^5}+\frac{8x^3}{3y^2}-\frac{8x^{\frac{5}{2}}y}{27}+\frac{x^2y^4}{81}$.
4. 12, or 3.
5. $\frac{a-2b}{2a-3b}$, or -1.
6. The volume of the 6-inch sphere is equal to the sum of the volumes of the other three spheres.

100.

1. $\frac{2\sqrt{a^2-x^2}}{x^2}$.
2. 5.414.
3. $x^2-2ax+a^2+5b=0$.
4. -8, or $-\frac{1}{8}$.
5. 3 hours, and 5 hours.

101.

1. $\frac{52}{a^4b^6c}$.
2. 5, or $-11\frac{1}{4}$.
3. $\frac{1}{4}(6+7\sqrt{3}+3\sqrt{7}+2\sqrt{21})$.
4. $x=4$ or $-\frac{1}{4}$, $y=1$ or $-3\frac{1}{4}$
5. 2.
6. $-\frac{2}{x-1}-\frac{10}{x-2}+\frac{18}{x-3}$.
7. $1-\frac{5x}{2}+\frac{5x^2}{2}-\frac{5x^3}{4}+\frac{5x^4}{16}-\frac{x^5}{32}$.

102.

1. $-\dfrac{5d^3}{3c^{m+2}}$.
2. $12+3\sqrt[3]{20}-12\sqrt[3]{3}-\sqrt[3]{180}$.
3. $x=4$, or -1; $y=1$, or -4.
4. 19, or 18.
5. $x=1$, or $\frac{1}{3}\sqrt[3]{3}$.
6. $1+\frac{1}{2}x+\frac{3}{8}x^2+\frac{5}{16}x^3+\frac{35}{128}x^4+\cdots\cdots$
$$+\frac{1\times3\times5\cdots\cdots(2r-1)}{2\times4\times6\cdots\cdots(2r)}x^r.$$

103.

1. $\sqrt[4]{\dfrac{a+b}{a-b}}$.
2. $x^{-4}-x^{-2}y^{-3}+y^{-6}$.
3. $\pm\frac{7}{5}$.
4. \$2704; \$103.
5. $-\frac{5}{1024}a^{-\frac{21}{4}}b^{18}$.
6. $x=\pm 2$, or $\pm\frac{4}{3}\sqrt{3}$; $y=\pm 6$, or $\pm\frac{10}{3}\sqrt{3}$.

104.

1. $\sqrt[6]{4000}$.
2. $\dfrac{16+4\sqrt{2}-4\sqrt{3}-\sqrt{6}}{13}$.
3. 1, or $1\pm2\sqrt{15}$.
4. 24 inches.
5. $715\,\dfrac{(2y)^4}{(3x)^{14}}$.
6. $\dfrac{1}{6(x+1)}-\dfrac{1}{2(x-1)}+\dfrac{4}{3(x-2)}$.

105.

1. $\sqrt[6]{a^2-b^2}$.
2. $\dfrac{am}{n-m}$, or $-\dfrac{am}{n+m}$.
3. ± 5, or $\pm 3\sqrt{2}$.
4. $x=7, -1$, or $3\pm\sqrt{-3}$.
$y=1, -7$, or $-3\pm\sqrt{-3}$.
5. -1.
6. $-1{,}400{,}000\,a^{12}$.

106.

1. $-\dfrac{21a}{bc^4}$.
2. $5+\sqrt{3}$.
3. $3, -\frac{1}{2}, \frac{1}{4}(5\pm\sqrt{1329})$.
5. 3, 5, 7.
6. $1+x-\frac{1}{2}x^2+\frac{1}{2}x^3-\frac{5}{8}x^4+\cdots\cdots$

107.

1. $460 + 11\sqrt{70}$.
2. $a^3 b^{\frac{4}{3}} c^{\frac{5}{6}}$.
3. $-8, -\frac{1}{8}$.
4. $x = \pm 5,\ y = \pm 2$.
5. 13, or 7.
7. $-\dfrac{10x^4}{243}$.

108.

1. $6\sqrt{3} - 13$.
2. 4, 8, 12, 16, 20, 24, ...
3. $x = 2$, or $\frac{3}{10}$; $y = \frac{1}{2}$, or $-\frac{6}{5}$.
4. 60.
5. $5\frac{2}{5}$.
6. $\dfrac{1}{4a^3(x-a)} - \dfrac{1}{4a^3(x+a)} - \dfrac{1}{2a^2(x^2+a^2)}$.
7. $-\dfrac{625x^3}{54}$.

109.

1. 24, 44, 80.
2. a, or $\dfrac{b}{a}(a+b)$.
3. 16 and 12.
4. $-\frac{47}{16}$.
5. $\frac{5}{7}$.
6. $1 + \frac{3}{2}x^2 + \frac{15}{8}x^4 + \frac{35}{16}x^6 + \frac{315}{128}x^8 + \cdots\cdots + \dfrac{3 \times 5 \times 7 \cdots\cdots (2r+1)}{2 \times 4 \times 6 \cdots\cdots (2r)} x^{2r}$.

110.

1. $a^{\frac{1}{2}} b^{\frac{1}{3}}$.
2. $\frac{1}{3}(14 - 4\sqrt{10})$.
3. 4, or -1.
4. $2650.
5. $\frac{2205}{4}$.
6. $\dfrac{2n(2n-1)\cdots\cdots(n+1)}{1 \times 2 \cdots\cdots n} x^n$.
7. $\dfrac{1}{5(x-7)} - \dfrac{1}{5(x-2)}$.

111.

1. 1.
2. $x + 3$.
3. 10 per cent.
5. 2, 4, 6, 8, 10.

112.

1. $x = \pm 1$, or ± 4; $y = \pm 3$, or ± 2.
2. 126 officers, 4000 men.
3. 1, 3, 5, 7, ...
4. $x = 3,\ -\frac{11}{2}$, or $\dfrac{-5 \pm \sqrt{33}}{4}$.
5. 21.

113.

1. $x = \pm 2$, or $\pm 3\frac{1}{3}$; $y = \pm 3$, or $\pm 2\frac{1}{3}$.

2. $\frac{22}{15}\sqrt{15}$. 3. $x = -3\frac{43}{67}$. 5. 15 inches.

114.

3. $\dfrac{2x^2 - 7xy + 5y^2}{x^2 - 5xy + 3y^2}$. 4. $\frac{3}{2}$. 5. 1,556,100.

115.

2. 2520. 3. $x = \dfrac{1 \pm \sqrt{5}}{2}$.

4. The means are 11, 16, 21, ·····; and the sum is 402.

5. 200 miles.

116.

1. 3, 6, 9, 12, ·····

2. $x = \dfrac{a-b}{2} \pm \frac{1}{2}\sqrt{a^2 + 2ab - 3b^2}$, $y = \dfrac{a-b}{2}$
$\mp \frac{1}{2}\sqrt{a^2 + 2ab - 3b^2}$; or $x = y = 0$, or $a + b$.

5. The scale of 9.

117.

3. $x = 1, -3, 3, -1$; $y = 2, 10, -10, -2$.

4. 2. 5. 0. 6. 16 inches.

118.

1. $2\frac{1}{4}, 4\frac{1}{4}, 6\frac{1}{4}$, ····· 3. $x^4 + 4x^2 - 96 = 0$.

2. $3, \frac{3}{4}, \frac{3}{16}$, ·····; or $1, \frac{3}{4}, \frac{9}{16}$, ····· 4. \$1250.

5. $x = \dfrac{3b^2 \pm \sqrt{3b(4a^3 - b^3)}}{6b}$, $y = \dfrac{-3b^2 \pm \sqrt{3b(4a^3 - b^3)}}{6b}$.

119.

1. $\pm 2a$, or $\pm a\sqrt{-6}$. 4. $y = \dfrac{16}{x^2} + \dfrac{4}{x} + 1$.

2. $7, -4, \frac{1}{2}(3 \pm \sqrt{349})$. 5. 6, 54.

120.

3. $2\frac{1}{7}$. **4.** 15, 3. **5.** 16.

121.

1. 288. **2.** 9, 12; or 1, -4. **3.** $b^2 = 5ac$.

4. $x = \frac{1}{2}\left(a \pm \sqrt{\frac{4b - a^2(2+m)}{2-m}}\right)$, $y = \frac{1}{2}\left(a \mp \sqrt{\frac{4b - a^2(2+m)}{2-m}}\right)$.

5. 25 cents.

122.

1. $x = \frac{-ac \pm \sqrt{a^2bc + bd - cd}}{b-c}$, $y = \frac{-ab \pm \sqrt{a^2bc + bd - cd}}{b-c}$.

If $b = c$, $x = y = \frac{0}{0}$ or ∞.

2. $2a^2$. **4.** $\frac{28}{125}a^{-\frac{14}{5}}b^3x^3$.

3. $-\frac{9}{2(x-1)} - \frac{20}{x-3} + \frac{61}{2(x-5)}$. **5.** 2520.

123.

1. 25.

2. 3, 6, 9, ...

3. 18 inches.

4. $x = \pm\sqrt{3}$, or ± 2;
$y = 0$, or ± 1.

5. 30,240.

124.

1. 100.

2. 2, 8, 32, 128; or $16\frac{7}{8}$, $28\frac{1}{8}$, $46\frac{7}{8}$, $78\frac{1}{8}$.

3. 5, $3 \pm \sqrt{2}$. **5.** 814.

4. $x = 3, -\frac{2}{3}$, $y = 2, \frac{1}{3}$. **6.** 10 and $10\frac{1}{2}$ miles per hour.

125.

1. 25. **2.** $\sqrt[3]{4}$, $\sqrt[3]{2}$, 1, $\sqrt[3]{\frac{1}{2}}$, $\sqrt[3]{\frac{1}{4}}$. **4.** Equal amounts.

5. $-\frac{m(m+1) \cdots\cdots (m+8)}{1 \times 2 \cdots\cdots 9} \times x^{-(m+9)} \times y^9$.

126.

2. $\pm \dfrac{a(m+n)}{\sqrt{2m^2+2n^2}}$. **5.** 1, 3, 9, 15; or 16, 8, 4, 0.

3. $x = \pm 10,\ y = \pm 5$. **6.** $123{,}410x^{20}$.

4. $\frac{4}{5}, \frac{3}{5}, \frac{2}{5}, \frac{1}{5}, 0, -\frac{1}{5}, -\frac{2}{5}, -\frac{3}{5}, -\frac{4}{5}$.

127.

1. $9 + \frac{11}{3}\sqrt{6}$. **3.** 12, or 8.

2. $x = 8$, or 4; $y = 4$, or 8. **4.** 5, 7, 9, 11, 13. **5.** 16.

6. $x = -\frac{1}{2}(y-1) + \frac{3}{8}(y-1)^2 - \frac{9}{16}(y-1)^3$
$+ \frac{135}{128}(y-1)^4 - \ldots\ldots$

128.

1. $6 - 6\sqrt[3]{6}$. **4.** 6, 8, 10. **5.** 1536.

2. 1, or $\frac{1}{3}\sqrt[3]{3}$. **6.** $10 \times 5^9 \times x^9$.

3. 240 feet, very nearly. **7.** Septenary.

129.

1. $a,\ \sqrt{a^2-b^2},\ \dfrac{a^2-b^2}{a}$. **2.** 24.

3. $x = 5, -3$, or $1 \pm \frac{1}{2}\sqrt{-88}$; $y = 3, -5$, or $-1 \pm \frac{1}{2}\sqrt{-88}$.

4. $l = -\dfrac{d}{2} \pm \sqrt{\left(\dfrac{d}{2} - a\right)^2 + 2ds}$.

5. $\dfrac{a^4}{81} + \dfrac{8a^5x}{243} + \dfrac{40a^6x^2}{729} + \dfrac{160a^7x^3}{2187} + \dfrac{560a^8x^4}{6561}$.

6. $\dfrac{6}{7(x-5)} - \dfrac{1}{5(x-3)} + \dfrac{12}{35(x+2)}$. **7.** $\underline{|6} \times \underline{|3}$.

130.

1. $\frac{1}{10}$. **4.** 5 days. **5.** 12.

2. ± 5, or $\pm 4\sqrt{2}$. **6.** $-2,\ \infty,\ 2,\ 1,\ \frac{2}{3}$.

3. 8 feet; 4 feet. **7.** $6a^{-14}x^{10}$.

131.

1. $\frac{1}{2}(3 \pm \sqrt{5})$, or $\frac{1}{6}(9 \pm \sqrt{-83})$.
2. $x = \pm 2$, or $\pm \frac{4}{3}\sqrt{3}$; $y = \pm 6$, or $\pm \frac{10}{3}\sqrt{3}$.
3. 8.
5. $1 + 3x + 4x^2 + 7x^3 + 11x^4 + 18x^5 + \cdots\cdots$
4. 10, 12, 15.
6. 84.

132.

1. $\frac{1}{7}(3 + 2\sqrt[3]{20} - 2\sqrt[3]{50})$.
5. 16.
2. 16, or $(-\frac{11}{5})^4$.
6. $\underline{\vert 7} \times \underline{\vert 4}$.
3. $x = (a \pm b)$, $y = (a \mp b)$.
4. 3, $\frac{10}{3}$, $\frac{11}{3}$, $\cdots\cdots$; and 5, $\frac{16}{3}$, $\frac{17}{3}$ $\cdots\cdots$

133.

1. $x = \frac{5}{2}$, or $-\frac{3}{2}$; $y = 3$, or -5.
2. 78 miles.
3. $\frac{3n(n+3)}{8}$.
4. $\frac{8}{21}$.

5. $$\frac{3}{2(x-6)} - \frac{3}{5(x-3)} + \frac{1}{10(x+2)}.$$

6. $$\frac{\underline{\vert 12}}{\underline{\vert 4}\,\underline{\vert 8}} \times \frac{\underline{\vert 8}}{\underline{\vert 3}\,\underline{\vert 5}}; \quad \frac{\underline{\vert 12}}{\underline{\vert 4}\,\underline{\vert 8}} \times \frac{\underline{\vert 7}}{\underline{\vert 2}\,\underline{\vert 5}}.$$

134.

1. 4.
4. 2, 5, 8, 11.
2. 5, or -30.
5. 2, 8, 32; or -2, -8, -32.
3. $x = 9, 3, -6, -9$; $y = 3, 9, -9, -6$.

6. $$-\frac{7 \times 3^5}{2 \times 4^8} \times \frac{y^{\frac{5}{3}}}{a^9 c^{\frac{7}{6}} x^{\frac{2}{3}}}.$$

135.

1. $\sqrt[36]{524288}$.
2. $\left(\frac{a \pm \sqrt{a^2 + 4b}}{2}\right)^{\frac{2n}{p}}$.
3. $\frac{3}{2}\sqrt{\frac{a}{g}}$.
4. 49, 1
5. $x = y - 1 + 2(y-1)^2 + 7(y-1)^3 + 30(y-1)^4 + \cdots\cdots$
6. $6\,\underline{\vert 4}\,\underline{\vert 4}$.

136.

1. $x = \pm\sqrt{\frac{a}{2b}},\ y = \pm\sqrt{\frac{b}{2a}}.$
2. 64.
3. 16, 24, 36, ...
5. $\frac{\underline{\lvert 36}}{\underline{\lvert 5}\ \underline{\lvert 31}};\ \frac{\underline{\lvert 35}}{\underline{\lvert 4}\ \underline{\lvert 31}}.$
6. $x = y + \frac{y^2}{\underline{\lvert 2}} + \frac{y^3}{\underline{\lvert 3}} + \cdots\cdots$

137.

1. $1,\ -\frac{1}{2} \pm \frac{1}{2}\sqrt{-3}.$
2. $x = 5$, or $-3\frac{3}{4}$; $y = 20$, or -15.
3. 33.
4. 194.
5. 44,330.
6. 123,225.

138.

1. $x = 0$, or $\frac{1}{2b}[a + 2 \pm \sqrt{a^2 - 4}]$;
   $y = 0$, or $\frac{1}{2b}[a + 2 \mp \sqrt{a^2 - 4}]$.
2. At 7 hours 48 minntes, the first time; and then at intervals of 2 hours 24 minutes.
3. $\frac{12}{25}$.
4. 44,100.
5. $x = y - y^2 + y^3 - 4y^5 + 14y^6 - 30y^7 + \cdots\cdots$

139.

1. $x = \frac{1}{2}a[\pm\sqrt{-1} \pm \sqrt{3}],\ y = \frac{1}{2}a[\pm\sqrt{-1} \mp \sqrt{3}].$
2. $10;\ \frac{\underline{\lvert 20}}{\underline{\lvert 10}\ \underline{\lvert 10}}.$
3. $\frac{2072}{3125}$.
4. 3, 2, 1, 4; or 1, 3, 3, 3.
5. $x = \pm 1$, or $\frac{3 \pm \sqrt{5}}{2}$.
6. $\frac{721}{228}$.

140.

1. They are reciprocals.
2. $x = \pm 2$, or $\pm\sqrt{-1}$; $y = \mp 1$, or $\mp 2\sqrt{-1}$.
3. 2.00498.
4. $\frac{1 + 3x}{(1 - x)^2}$.
5. $2 - x - x^2 + 2x^3 - \cdots\cdots$
6. $x = 9,\ y = 2$.

141.

2. 3, -6, $\frac{4}{5}$.
3. $\log a + \frac{m}{n}\log c + \operatorname{colog} b + \frac{1}{2}\operatorname{colog} d$.
4. 146.
5. 15.68.
6. 1.0788.

142.

2. 2, -4, 12, $-\frac{5}{7}$.
3. $9.00000 - 10$.
4. $5.81238 - 10$.
5. 0.02524.
6. 0.01987.

143.

2. -3, -4, $\frac{2}{3}$.
3. 0.87174.
4. 0.71426.
5. 1.537.

144.

1. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{5}{6}$, $-\frac{4}{3}$.
2. $\frac{1}{4}$.
3. 31.1.
4. 39.99.
5. 2.57.
6. 10 years.

145.

1. 3, 2, -4.
2. $\frac{1}{1000000}$, $\frac{1}{243}$, 243.
3. $\frac{3}{10}$.
4. 0.04.
5. 0.0001129.
6. 5.593.

146.

1. 2, 4, -3.
2. 0.5646.
3. 0.1941.
4. 1.703.
5. 0.585.

3. $613.70.

147.

1. 6.
2. $\frac{1}{3}$.
3. 0.0001404.
4. 0.2929.
5. 4.429.
6. 36.7 years.

148.

1. -4.
2. 0.4266.
3. 2.94444, 3.13549, 3.36730, 3.43399.
4. 8.058.
5. $29,979.30.
6. 9.588.

149.

2. $1, -1, 3, \frac{7}{5}, m-n, -\frac{n}{m}$.

3. 1.816.

4. 1.067.

5. 0.69897, 1.04139.

150.

1. $1, 5, -1, 0, n, \frac{m-n}{m}$.

2. 139.8.

3. 0.2372.

4. 1.18329.

5. 1.60529.

6. $66\frac{5}{7}$ years.

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